



Heterogeneous expectations and the distribution of wealth



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ABSTRACT

This study examines the extent to which heterogeneity of expectations affects wealth distribution, through the use of a standard heterogeneous agent model with uninsured idiosyncratic risk and aggregate uncertainty. A simple stylized model of heterogeneous expectations is considered to demonstrate that the impact of expectations' heterogeneity on wealth inequality depends nonlinearly on the level and persistence of expectations' dispersion. It is also shown that the heterogeneity of expectations generated by the empirically validated learning-from-experience model (Malmendier, Nagel, Q J Econ 2016) has a moderate but ambiguous impact on the distribution of wealth. The effect is sensitive to the calibration of the macroeconomic and learning parts of the model.

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1. Introduction

It is generally accepted that expectations play an important role in explaining many economic phenomena. A growing body of literature also emphasizes that agents' expectations are heterogeneous (see [Pesaran and Weale, 2006](#); [Hommes, 2011](#)). However, while most research has focused on the empirical validation or rationalization of the expectations' heterogeneity, less attention has been given to an examination of its consequences. This study intends to fill this gap by studying the role of heterogeneous expectations in shaping the distribution of wealth in the incomplete markets model with uninsured idiosyncratic risk and aggregate uncertainty of [Krusell and Smith \(1998\)](#).

The key problem in examining implications of the expectations' heterogeneity is the lack of a single, commonly accepted theoretical setup. Instead, several different models are considered in the literature, as discussed by [Pfafar \(2013\)](#), with possibly contradictory properties regarding the level and persistence of expectations' dispersion. The models cannot be thoroughly verified because of a scarcity of empirical evidence.

In order to overcome this difficulty, a general model of heterogeneous expectations is developed, which serves as an approximation of the various specifications considered in the literature. In this model, the level and persistence of the expectations' dispersion are specified exogenously. This tool is used to investigate theoretically how the interplay of these two characteristics affects wealth inequality.

Subsequently, the learning-from-experience model of heterogeneous expectations ([Malmendier and Nagel, 2016](#)) is analyzed to examine which channels are relevant in the empirically grounded model. The model was developed to match the

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observed heterogeneity in inflation expectations between agents of different age groups. Basically, this is a standard adaptive learning model with a heterogeneous and time-varying gain parameter. Its implications for wealth inequality and the underlying properties of the induced dispersion of expectations are studied.

This study is related to three areas of the literature. First, it builds on the literature that documents the heterogeneity of expectations. Several studies explore survey data and find evidence on the heterogeneity of expectations related to the stock market outlook (Vissing-Jorgensen, 2003), exchange rates (MacDonald and Marsh, 1996; Elliott and Ito, 1999; Dreger and Stadtmann, 2008), inflation (Mankiw et al., 2003; Capistran and Timmermann, 2009; Doornik et al., 2012; Pfajfar and Santoro, 2013), and other macroeconomic variables (Dreger and Stadtmann, 2008). Bryan and Ventkau (2001) and Gnan et al. (2011) also report significant variations in inflation expectations between different socio-economic groups. Pesaran and Weale (2006) and Hommes (2011) present a more detailed review of the survey-based evidence. Kurz (1994), Brock and Hommes (1998) and Branch (2004) have made theoretical contributions that suggest rational agents may hold heterogeneous expectations. Heterogeneity of expectations is usually rationalized by differences in forecasting models, information sets, and the capacity for information processing. Pfajfar (2013) presents a detailed review of the more recent contributions to this field.

The study contributes to the literature on the role of information in macro models with incomplete markets. Porapakkarm and Young (2008) consider a model where partially-informed agents must rely on the Kalman filter to extract estimates of the state of the economy based on observed prices. They find that an economy with partially-informed agents is characterized by considerably lower wealth concentration compared to an economy with fully-informed agents. The same model is studied by Graham and Wright (2010) who investigate properties of equilibrium. Cogley et al. (2014) study wealth dynamics in a model with partially- and fully-informed agents; they also show that under incomplete markets the learning agents accumulate wealth, while the fully-informed agents are driven to their debt limit.

This study is closely related to the work of Giusto (2014) who utilizes the Krusell-Smith model with an adaptive learning scheme to investigate the impact of boundedly-rational but homogeneous expectations on wealth distribution. His work demonstrates that once the perceived level of aggregate capital exceeds the equilibrium level, wealth inequality rises due to opposite effects on optimal consumption levels of capital-rich and capital-poor agents.

Finally, this study also adds to the literature on the sources of heterogeneity in wealth that was initiated in the influential studies of Aiyagari (1994) and Huggett (1993). Accounting only for heterogeneous employment histories, these early models do not generate a realistic distribution of wealth and income. A number of solutions have been proposed for this problem. Krusell and Smith (1998) propose a version that utilizes a heterogeneous, time-varying time preference. In this setup, more patient agents accumulate more wealth. However, the volatility of the discount coefficient prevents them from gathering all the wealth in the economy. The heterogeneity of the time preference is also examined by Hendricks (2007). Castaneda et al. (2003) and Chang and Kim (2006) utilize heterogeneous and time-varying labor productivity. Their models are calibrated to match income inequality and generate empirically reasonable wealth concentration. Quadrini (2000) and Cagetti and Nardi (2006) add entrepreneurs who accumulate a large share of wealth. Conversely, Castaneda et al. (2003) and De Nardi (2004) emphasize the importance of intergenerational transfers within the overlapping generations framework to explain the observed wealth inequality. Luo and Young (2009) and Suen (2014) posit a solution to the problem by accounting for wealth in a utility function, which represents the agents' demand for status. The recent work of del Rio (2015) takes into account heterogeneity in labor disutility and the market skills of agents. Finally, Bielecki et al. (2015) develops a model with rich ex ante heterogeneity of agents. An extensive survey of wealth inequality models is presented by De Nardi (2015).

The results of the study can be summarized as follows. The general model of heterogeneous expectations indicates that if the differences in expectations are persistent, then even a small heterogeneity would result in substantial wealth inequality. The reason for this effect is that heterogeneous expectations imply different saving rates for different agents. The agents who anticipate higher interest rates and lower wages have a tendency to save more, on average, than the agents who expect that the opposite will occur. If the differences in expectations persist, then the rise in wealth inequality can be significant. Conversely, if the agents frequently alter their expectations, then the wealth inequality can decrease. This is because the agents' expectations are almost identical, considering averages across time, and their decision rules are primarily driven by the expectations and to a smaller extent by their current employment status and the aggregate state of the economy. These two effects are amplified by a high dispersion of expectations.

It is also seen that the expectations' heterogeneity generated by the learning-from-experience model considerably affects the distribution of wealth. However, the sign and the magnitude of the impact are sensitive to the calibration of the macroeconomic and learning parts of the model.

The study is organized as follows. Section 2 contains a brief description of the workhorse model of Krusell and Smith. In Section 3, the general model of heterogeneous expectations is considered. Subsequently, the learning-from-experience model is examined. Section 5 concludes the paper.

2. The model

This study utilizes the standard Krusell–Smith heterogeneous agent model with uninsured idiosyncratic risk and aggregate uncertainty. Aggregate variables are denoted by capital letters, while individual characteristics are represented by small letters.

2.1. The decision problem of the agent

The economy is populated by a continuum of infinitely-lived agents. The agents are identical ex-ante, but they differ in terms of wealth ex-post because of different employment histories. The employment status of an agent is described by a random variable ϵ_t , where $\epsilon_t = 0$, if she is unemployed and $\epsilon_t = 1$, otherwise. Evolution of the labor market status is described by a two-state Markov chain. The employed agent earns a net wage $(1 - \tau_t)\bar{l}W_t$, where τ represents the tax rate, \bar{l} is the agent's labor supply, and W is the wage. The unemployed agent receives the unemployment benefit μW_t , where μ is the unemployment replacement rate. All agents earn interest on their beginning-of-period capital k_t that is equal to $(R_t - \delta)k_t$. In this case, R_t represents the interest rate and δ is the capital depreciation rate.

In every period t , the agent decides how to divide her current wealth between consumption and saving. The decision problem can be written as follows:

$$\max_{c_t, k_{t+1}} \mathbb{E}_t \sum_{h=0}^{\infty} \beta^h u(c_{t+h}) \text{ s.t.} \quad (1)$$

$$c_{t+h} + k_{t+1+h} = (1 - \delta + R_{t+h})k_{t+h} + [(1 - \tau_{t+h})\bar{l}\epsilon_{t+h} + \mu(1 - \epsilon_{t+h})]W_{t+h}, \quad (2)$$

$$k_{t+h} \geq \underline{k}, \quad h = 0, 1, \dots, \quad (3)$$

where \mathbb{E}_t is a conditional expected value operator and \underline{k} stands for a debt limit. A standard constant relative risk aversion (CRRA) utility function is used with γ representing a relative risk aversion coefficient:

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}. \quad (4)$$

2.2. The production sector

The production sector is comprised of one representative firm that hires capital and labor from the agents and produces a single consumption good according to the standard Cobb-Douglas technology:

$$Y_t = Z_t K_t^\alpha (\bar{L}_t)^{1-\alpha}, \quad (5)$$

where K and L represent aggregate capital and labor, respectively, Z denotes the aggregate productivity shock modeled by a two-state Markov chain, and α is the capital share parameter. Since the firm operates in a competitive market, the aggregate wage and interest rate are equal to the marginal products of both labor and capital, respectively:

$$W_t = (1 - \alpha)Z_t K_t^\alpha (\bar{L}_t)^{-\alpha}, \quad (6)$$

$$R_t = \alpha Z_t K_t^{\alpha-1} (\bar{L}_t)^{1-\alpha}. \quad (7)$$

2.3. Government

The specification of the government's role in the model follows the modification studied by [den Haan et al. \(2010\)](#). Instead of the lump-sum tax specification used by [Krusell and Smith \(1998\)](#), it is assumed that the government imposes taxes on labor to finance unemployment benefits. The government's budget is balanced every period, which implies that the tax rate τ_t is set according to the formula:

$$\tau_t = \frac{\mu u_t}{\bar{l} L_t}, \quad (8)$$

where u denotes the unemployment rate.

2.4. Calibration

The calibration procedure closely follows [Krusell and Smith \(1998\)](#). One period in the model corresponds to a quarter. The baseline calibration is summarized in [Table 1](#).

The standard values for the capital share ($\alpha = 0.36$) and its depreciation ($\delta = 0.025$) are used. The discount factor β is set to 0.99 to generate the realistic average interest rate level of 3.5% per annum. The risk aversion parameter γ is equal to 1. In this case, the CRRA utility function collapses to logarithmic utility. Finally, the time endowment \bar{l} is equal to 0.31.

The technology shock takes two values representing booms and recessions. The standard deviation of the shock is equal to 1%. The transition matrix of the Markov chain for the idiosyncratic and aggregate shocks is also taken from [Krusell and Smith \(1998\)](#). It is presented in [Appendix A](#).¹

¹ The appendices can be found on-line, as a supplementary material.

Table 1
The baseline calibration of the model

Parameter	Description	Value
α	capital share	0.36
β	discount coefficient	0.99
δ	capital depreciation rate	0.025
\bar{l}	time endowment	0.31
γ	relative risk aversion	1
Z_g	productivity shock in a boom	1.01
Z_b	productivity shock in a recession	0.99
μ	unemployment replacement rate	0.15
\underline{k}	debt limit	−4

Table 2
 b^{RE} coefficients for the baseline calibration

	b_{0b}^{RE}	b_{1b}^{RE}	b_{0g}^{RE}	b_{1g}^{RE}	$\log \bar{K}_{impl,b}^{RE}$	$\log \bar{K}_{impl,g}^{RE}$
Value	0.0786	0.9660	0.0898	0.9636	2.31	2.47

Unlike in [Krusell and Smith \(1998\)](#), the positive unemployment benefit μ is assumed. Following [den Haan et al. \(2010\)](#), it is equal to 0.15. The borrowing constraint is set to $\underline{k} = -4$ to match the average fraction of agents with negative net wealth in the United States, which is close to 10%.

The replacement rate of unemployment benefits equal to 15% is significantly lower than the legal replacement rate for most of the states in the United States, which is 50% of the lost income. However, the benefits are paid only for six months at most, whereas, in the model, the limit does not apply and unemployment lasts six months on average. Moreover, an upper limit on the absolute value of a benefit exists in most states. Nonetheless, the higher unemployment benefits of 40% together with the appropriately adjusted borrowing constraint is also considered. The main results are not qualitatively different from those for the baseline calibration and are presented in Appendix F.

2.5. Rational expectations

In the following notation, the time subscripts are omitted and the next period’s variables are denoted with primes. Using the same approach as [Krusell and Smith \(1998\)](#), the decision problem of the consumer can be approximated as:

$$v(k, \epsilon, K, Z) = \max_{c, k'} \{u(c) + \beta \mathbb{E}v(k', \epsilon', K', Z')\} \quad \text{s.t.} \tag{9}$$

$$k' + c = (1 - \delta + R)k + [(1 - \tau)\bar{l}\epsilon + \mu(1 - \epsilon)]W, \tag{10}$$

$$\log K' = b_{0b} + b_{1b} \log K \quad \text{if } Z = Z_b, \tag{11}$$

$$\log K' = b_{0g} + b_{1g} \log K \quad \text{if } Z = Z_g, \tag{12}$$

$$k, k' \geq \underline{k}, \tag{13}$$

where v denotes the value function. The parameters $\mathbf{b} = [b_{0b}, b_{1b}, b_{0g}, b_{1g}]$ govern the agent’s expectations regarding capital evolution. [Krusell and Smith \(1998\)](#) proposed an iterative procedure for determining the values of the parameters that makes the expectations almost fully rational. As a result, the agents’ perceived law of motion for the aggregate capital stock almost coincides with its actual dynamics. However, as already noted by [Giusto \(2014\)](#) and [Cozzi \(2015\)](#), among others, the long-run average of the wealth distribution in the model is primarily determined by the implied expected aggregate capital stock defined as follows:

$$\log \bar{K}_{impl,j} = \frac{b_{0j}}{1 - b_{1j}}, \quad j = \{b, g\}. \tag{14}$$

Hereafter, the expectations are defined by referring to $\bar{K}_{impl,j}$, instead of using the vector \mathbf{b} .

In [Tables 2](#) and [3](#), the main characteristics of the model for the baseline calibration are summarized.

Table 3
Characteristics of the model for the baseline calibration

	\bar{K}	\bar{R}	\bar{W}	$sd(K)$	$sd(R)$	$sd(W)$	Gini
Value	10.96	0.035	2.37	0.280	0.001	0.021	0.548

The results are obtained from a simulation with 10,000 agents and 30,000 periods where the first 5000 periods are treated as a burn-in sample; “ $sd(X)$ ” denotes a standard deviation of time series of a variable X ; the normalized Gini coefficient is calculated according to [Chen et al. \(1982\)](#) and [Berrebi and Silber \(1985\)](#) formula.

3. General model of heterogeneous expectations

In this section, the impact of the level and the persistence of the cross-sectional dispersion of expectations on the wealth distribution is studied theoretically. A flexible, stylized model of heterogeneous expectations is developed that accounts for various possible properties of the expectations’ differences.

3.1. Specification

In order to introduce the heterogeneity of expectations into the model, it is assumed that the agents do not share the same rational expectations (RE). Instead, they randomly switch between two alternative forecasting rules:

$$\log K' = b_{0j}^{(I)} + b_{1j}^{(I)} \log K, \quad j = \{b, g\}, \quad (\text{rule I})$$

$$\log K' = b_{0j}^{(II)} + b_{1j}^{(II)} \log K, \quad j = \{b, g\}. \quad (\text{rule II})$$

The rules are characterized by different values of $\bar{K}_{impl,j}$. Specifically, the agents who use the rule I expect that the aggregate capital stock will be lower than the equilibrium value, and the parameters $b_{0j}^{(I)}$ and $b_{1j}^{(I)}$ are set in order that $\bar{K}_{impl,j}^{(I)} = (1 - \Delta)\bar{K}_{impl,j}^{eq}$, $j = \{b, g\}$. The agents who apply the rule II predict that the aggregate capital stock will be higher than the equilibrium value, and the parameters $b_{0j}^{(II)}$ and $b_{1j}^{(II)}$ are set in order that $\bar{K}_{impl,j}^{(II)} = (1 + \Delta)\bar{K}_{impl,j}^{eq}$, $j = \{b, g\}$. The parameter Δ measures the relative cross-sectional dispersion of expectations. The details on the selection of the parameters defining the forecasting rules are presented in Appendix B.

The values of $\bar{K}_{impl,j}^{eq}$ define the implied levels of the aggregate capital stock for the actual law of motion of aggregate wealth under the heterogeneous expectations. These levels do not exactly coincide with the implied levels for the RE actual law of motion. When the cross-sectional dispersion of expectations and their persistence are higher, the discrepancy between the two laws of motion increases. As shown below, a high dispersion of expectations combined with their high persistence significantly increases the model’s aggregate capital stock. If the heterogeneous expectations are defined in relation to the fixed RE expectations, then the absolute level of the actual expectation bias would become lower for the group that uses the rule II compared to the users of rule I.

The agents are allowed to change their expectations randomly. Therefore, every period there is a fraction p of the agents who switch their forecasting rules. However, the possible switch between the rules is not taken into account in the decision problem (9)–(13).

An iterative procedure, analogous to the Krusell–Smith algorithm (described in Appendix B) is used to find the implied levels of the aggregate capital stock $\bar{K}_{impl,j}^{eq}$ and the actual law of motion of aggregate capital under the heterogeneous expectations. The additional simulations show that the correction discussed above significantly affects the aggregate capital stock in the model. However, the wealth inequality measures remain virtually unchanged.

It should be noted that the expectations of the agent are biased in the same direction, regardless of the business cycle phase. If the agent’s expected aggregate capital is $\bar{K}_{impl,j}^{(I)}$, it means that she expects it to be lower by $(100 \cdot \Delta)\%$ than the RE value in both the bad and good state of the economy.

3.2. Calibration

Because of the lack of direct empirical evidence on the cross-sectional heterogeneity of average wealth expectations several combinations of reasonable values of Δ and p are considered. Specifically, it is assumed that $\Delta = \{0.02, 0.055, 0.4\}$. The first value is close to the relative standard deviation of the time series of aggregate capital in the RE version of the model. In this case, the wage and interest rate expectations are biased just by 0.72% and 1.28%, respectively, which appear to be very low biases compared to what is observed for inflation expectations. The second value is equal to the standard deviation of the time series of aggregate capital estimated by [Giusto \(2014\)](#). The implied bias in the production factor prices is almost three times higher, but it is still relatively low. The last choice generates a considerably higher anticipated wage and interest rate dispersion of approximately 20%.

Table 4
Results for the general model of heterogeneous expectations

ψ	2	2	2	100	100	100	200	200	200	∞	∞	∞	RE
Δ	0.02	0.055	0.4	0.02	0.055	0.4	0.02	0.055	0.4	0.02	0.055	0.4	
\bar{K}	10.98	10.98	11.07	10.98	11.02	12.12	10.99	11.06	12.80	11.12	11.40	14.55	10.96
$sd(K)$	0.287	0.287	0.298	0.286	0.284	0.291	0.285	0.281	0.291	0.268	0.260	0.278	0.280
Gini	0.426	0.213	0.074	0.512	0.531	0.630	0.550	0.608	0.678	0.646	0.630	0.599	0.548
top 1 [%]	4.5	2.2	1.3	5.1	4.9	4.9	5.3	5.6	4.6	3.5	3.0	2.4	5.9
top 10 [%]	27.9	17.2	12.1	31.8	31.8	35.2	33.4	35.9	36.8	29.5	26.9	23.8	34.7
neg. wealth [%]	3.3	0.1	0.0	9.1	12.6	25.3	13.1	20.5	33.7	50.0	50.0	50.0	9.7

The results are obtained from a simulation with 10,000 agents and 30,000 periods; the first 5000 periods are treated as a burn-in sample; “sd(K)” denotes a standard deviation of the time series of aggregate capital; “top 1” and “top 10” represent average shares of total wealth held by the wealthiest 1% and 10% of agents, respectively; “neg. wealth” represents the fraction of agents with negative wealth; the normalized Gini coefficient is calculated according to [Chen et al. \(1982\)](#) and [Berrebi and Silber \(1985\)](#) formula.

The following four different values are used for p : 0.5, 0.01, 0.005, and 0. The value $p = 0.005$ implies that the average duration $\psi = 1/p$ of a state is 50 years (200 quarters). This number is used by [Krusell and Smith \(1998\)](#) in their stochastic beta model for the extreme values of β . The two extreme cases correspond to very frequent switches of expectations (every $\psi = 2$ periods, on average) and fixed expectations ($\psi = \infty$).

3.3. Results

Several moments of the wealth distribution are presented in [Table 4](#). The results clearly show that the persistence parameter ψ determines whether wealth inequality is lower or higher compared to the RE case. The frequent switches of expectations ($\psi = 2$) reduce wealth concentration. Conversely, the inequality measures rise if the differences in expectations are persistent ($\psi = 100$ and $\psi = 200$). These effects are amplified if the dispersion of expectations is higher. For example, if $\psi = 2$ and $\Delta = 0.4$, then the decrease in inequality is higher than in the case of $\Delta = 0.02$.

Few exceptions from the general rules can be observed. For example, if $\psi = \infty$, then increasing Δ does not raise wealth inequality. Similarly, if ψ rises from 200 to ∞ , then some measures of wealth inequality (several wealth shares of the wealthiest agents and the Gini coefficient for $\Delta = 0.4$) do not grow as expected. These effects occur because of the granularity of the model with only two forecasting rules and almost fully disappear as more rules are considered.²

As mentioned in the previous section, the aggregate capital stock rises considerably as the dispersion of expectations becomes higher and more persistent. The high sensitivity of the fraction of agents with negative wealth to changes in the parameters governing the expectation formation process should also be noticed.

3.4. Discussion

This section discusses the mechanisms that lead to the observed differences in wealth concentration between the heterogeneous expectation model and its RE counterpart. First, the rise in wealth inequality under the highly persistent expectations’ differences is analyzed.

3.4.1. The highly persistent expectations

[Fig. 1](#) illustrates the wealth distribution in the model under extreme calibration $\Delta = 0.4$ and $\psi = \infty$. With permanent differences in the expectations, the two groups of agents have completely different wealth holdings. The agents who expect a low level of aggregate capital compared to the equilibrium level also anticipate lower wages and higher interest rates in the next period. Therefore, they tend to save more than under rational expectations. The opposite is true for the second group. Although there are very small differences in the saving rates, they are highly persistent and lead to significant wealth inequality.

The high sensitivity of the agents’ capital stock to the factor prices results from the weak general equilibrium self-correcting mechanism that works well in the standard [Krusell–Smith](#) model with homogeneous expectations. In their setup, all the agents decrease their saving rates when they expect a surge in the aggregate capital stock. This leads to a decline in aggregate capital stock and increase in saving rates. This is not the case with heterogeneous expectations model. A drop in the saving rate of one group is offset by a rise in the second one, thereby leading to a slight change in the aggregate capital stock. The differences in the saving rates remain relatively stable for longer periods of time, which results in a higher concentration of wealth.

3.4.2. The frequent switches of expectations

If the agents switch their expectations frequently, then wealth inequality becomes smaller when compared to the RE version. At first sight, this may appear to be a somewhat counter-intuitive result, but it can be easily explained. In the RE

² The results for the model with more forecasting rules are available upon request.

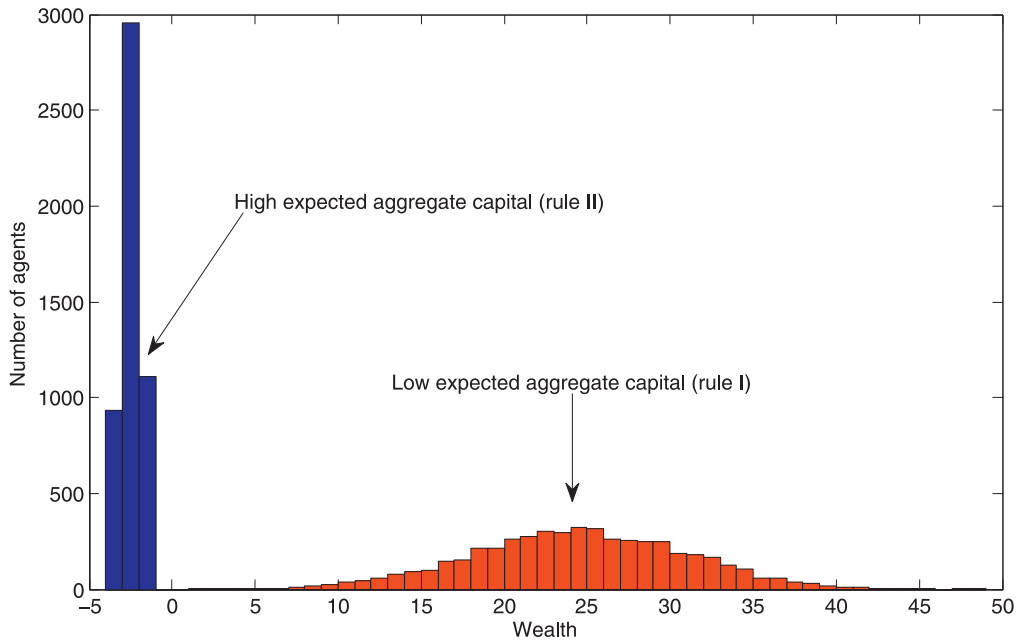


Fig. 1. Wealth distribution with the highly persistent expectations ($\Delta = 0.4, \psi = \infty$)

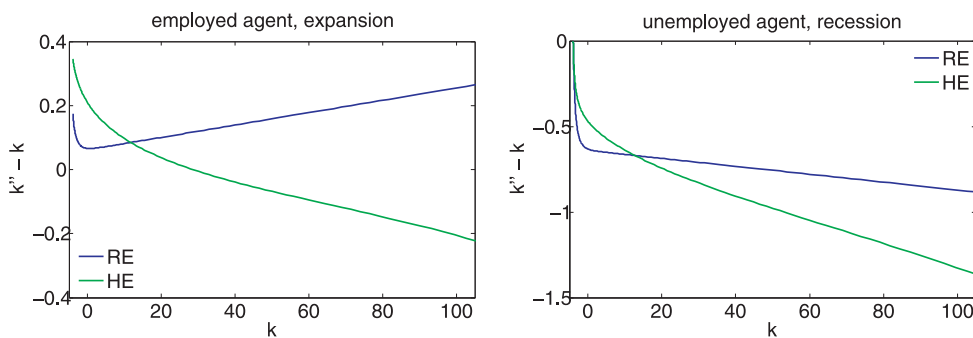


Fig. 2. Decision rules under rational (RE) and heterogeneous (HE), rapidly-switching expectations ($\Delta = 0.4, \psi = 2$). In the plots, the two-period-ahead decision rules are depicted. They show the amount of capital saved during the two periods by agents having a different initial wealth level k . In the case of RE, the agents always use the standard RE decision rule. In the case of HE, they use the rule I in the first period and the rule II in the second period. The left plot assumes that the agents are employed and the economy is in expansion mode in both periods. In the right plot, the opposite assumptions are made. In both cases, the aggregate capital stock is always equal to 11.

model, the agents become rich because of having a job for a long time. Subsequently, they quickly accumulate wealth during expansions and slowly dissave during recessions. However, agents often dissave even during expansions if their expectations are not fully rational and change frequently. This can happen when the agents' anticipated interest rate is lower than the RE interest rate, and they no longer find it optimal to accumulate wealth further. Such situations usually occur when the agents' expectation bias is large.

This effect is illustrated in Fig. 2 where the case $\Delta = 0.4$ and $\psi = 2$ is considered. The two-period-ahead decision rules are depicted that show a change in the agents' capital stock for individuals who hold rational expectations for two periods and those who expect that the aggregate wealth will be much lower and then much higher than the RE level.³ As already mentioned, under rational expectations, the employed agents accumulate wealth during expansions, regardless of their initial capital stock. At the same time, the agents with heterogeneous expectations increase their capital stock only if it is lower than 30. Additionally, the poorest agents save more under heterogeneous expectations than under rational expectations. Similar effects are observed for the unemployed agents during recessions, as illustrated in the right plot, and other

³ The order of the bias direction has no impact on the results. The decision rules of the agents who first expect a higher and then a lower aggregate wealth are virtually the same.

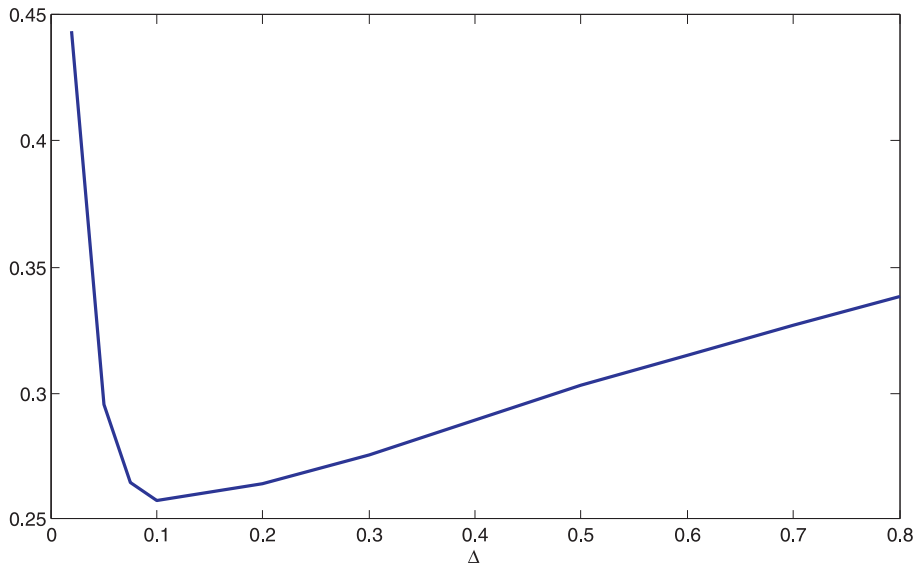


Fig. 3. The relationship between the Gini coefficient and the level of the expectations' dispersion parameter Δ for $\psi = 10$

combinations of individual and aggregate shocks. As a result, wealth inequality under heterogeneous expectations is much lower than in the case of RE.

3.4.3. The role of expectations' dispersion parameter Δ

From Table 4, it is clear that the level of expectations' dispersion amplifies the effects generated by the persistence of expectations' heterogeneity. The higher the Δ , higher is the wealth inequality in the case of highly persistent expectations and lower is the wealth concentration under frequently switching expectations. However, in the latter case, this picture is not complete.

On the one hand, expectations' heterogeneity can reduce the inequality resulting from different employment histories, as shown above. On the other hand, it generates additional wealth inequality that arises from differences in the decision rules. A sufficiently large dispersion of expectations allows the latter effect to dominate, wherein the aggregate wealth inequality stops declining with an increase in Δ . This is illustrated in Fig. 3.

In this case, the expectations switch frequently, and hence a rise in Δ leads to an initial decline in wealth inequality. The Gini coefficient reaches its minimum for $\Delta = 0.1$ and then starts to increase slightly.

4. The learning-from-experience model of heterogeneous expectations

In this section, the distribution of wealth in the learning-from-experience model of heterogeneous expectations (Malmendier and Nagel, 2011, 2016) is studied. This is a standard adaptive learning model with a heterogeneous and time-varying gain. It is assumed that the gain of an individual is a decreasing function of her age. This reflects the empirical fact that expectations of young agents react strongly to surprises compared to the expectations of older individuals because the lives of young agents are largely composed of new experiences. Therefore, early-life experiences play an important role in shaping agents' expectations than experiences in the later life. The heterogeneity of expectations occurs because agents who are born in different phases of a business cycle hold different beliefs throughout their lifetimes.

It is assumed that the agents enter the labor market at the age of 21 and die at the age of 100, at most. The length of life is stochastic and the number of agents is constant. The dead agents are immediately replaced by their descendants who also inherit their total wealth. The infinite horizon setup regarding the agents' decision rules considered in the previous sections is maintained by assuming that the agents are perfectly altruistic. Formally, the decision problem of an agent i of age a is written as:

$$v(a, k, \epsilon, K, Z, \hat{\mathbf{b}}_i) = \max_{c, k'} \{u(c) + \beta \mathbb{E}(s_a v(a + 1, k', \epsilon', K', Z', \hat{\mathbf{b}}_i)) + (1 - s_a) v_d(1, k', \epsilon', K', Z', \hat{\mathbf{b}}_i)\}, \tag{15}$$

subject to (10)–(13). The vector $\hat{\mathbf{b}}_i$ contains the parameters defining the agent's expectations that evolve according to the standard formulas (19) and (20) that are defined below, s_a stands for the surviving probability from age a to age $a + 1$ that is taken from the United States life tables for the year 2013 published by the United States Social Security Administration,

and v_d represents a value function of a descendant who faces the same decision problem. This specification implies that:

$$v(a, k, \epsilon, K, Z, \hat{\mathbf{b}}_t) = v_d(a, k, \epsilon, K, Z, \hat{\mathbf{b}}_t) = v(a + 1, k, \epsilon, K, Z, \hat{\mathbf{b}}_t) \equiv v(k, \epsilon, K, Z, \hat{\mathbf{b}}_t) \quad (16)$$

and the decision problem is independent of age.⁴

4.1. The setup of the learning mechanism

It is assumed that the agents are not aware of the coefficients of the RE law of motion for aggregate capital as defined in (11) and (12). Instead, they estimate them using the correctly specified models:

$$\log K_t = \hat{b}_{0b} + \hat{b}_{1b} \log K_{t-1} \quad \text{if } Z_{t-1} = Z_b, \quad (17)$$

$$\log K_t = \hat{b}_{0g} + \hat{b}_{1g} \log K_{t-1} \quad \text{if } Z_{t-1} = Z_g. \quad (18)$$

The parameters of each equation are estimated separately by the recursive, weighted least squares defined by the following recursion:

$$\mathbf{S}_{t,i,j} = \mathbf{S}_{t-1,i,j} + g_{t,i} (\mathbf{x}'_{t-1} \mathbf{x}_{t-1} - \mathbf{S}_{t-1,i,j}), \quad (19)$$

$$\hat{\mathbf{b}}_{t,i,j} = \hat{\mathbf{b}}_{t-1,i,j} + g_{t,i} \mathbf{S}_{t,i,j}^{-1} \mathbf{x}_{t-1} (y_t - \mathbf{x}'_{t-1} \hat{\mathbf{b}}_{t-1,i,j}), \quad (20)$$

where $\hat{\mathbf{b}}_{t,i,j} = [\hat{b}_{0j,t,i}, \hat{b}_{1j,t,i}]'$; $y_t = \log K_t$; $\mathbf{x}_t = [1, \log K_{t-1}]'$, and the indexes i and j identify the individuals and the aggregate state of the economy, respectively.⁵

4.1.1. Dynamics of the gain parameter

The gain parameter $g_{t,i}$ of an individual i evolves according to the following formula (Malmendier and Nagel, 2016):

$$g_{t,i} = \begin{cases} 1 & \text{if } t - s(i) < \theta \\ \frac{\theta}{t - s(i)} & \text{if } t - s(i) \geq \theta \end{cases} \quad (21)$$

where $s(i)$ denotes the period when the individual i is born⁶ and θ governs the importance of the early-life experience for determining agents' expectations. The higher the θ , the more the weight is put on the data observed in the early life.

Formula (21) implies that $g_{t,i} = 1$ in the first θ quarters of life, which means that only the most recent observation is used for forming the expectations. Subsequently, the matrix $\mathbf{S}_{t,i,j}$ becomes singular and non-invertible. In this case, the stochastic gain algorithm (Evans and Honkapohja, 2001) is employed, which simply does not account for \mathbf{S}^{-1} in the estimates updating Eq. (20). This setup also implies that the agents from the same cohort share the same expectations as they completely disregard the inherited experience and use the same past data to form their expectations.

4.1.2. Calibration of the gain parameter

Malmendier and Nagel (2016) estimated the value of θ to match the cross-sectional heterogeneity of the inflation expectations in the United States. The parameter and standard error estimates obtained by the authors are equal to 3.044 and 0.233, respectively. Alternative specifications of the econometric model yield the estimates close to 4, with the standard errors equal to 0.5.

In this study, the standard assumption made from the adaptive learning literature is that the agents forecast the log aggregate capital stock. Given the linear relationship between $\log K$ and $\log R$, it is equivalent to predicting the real interest rate. Unfortunately, there is no reliable data on the heterogeneity of the real interest rate expectations that could be directly used in calibrating the gain parameter θ . Therefore, the baseline value for the gain parameter follows the estimate obtained by Malmendier and Nagel (2016) and is set to $\theta = 3$ assuming that the weighting scheme of the past data for the real interest rate prediction should be close to the one employed for forecasting the inflation rate since the two variables are characterized by similar time-series properties. Moreover, the value of $\theta = 3$ is consistent with the joint estimate of the learning scheme for inflation, nominal interest rate, and output gap obtained by Milani (2007), as shown by Malmendier and Nagel (2016). Nonetheless, given the uncertainty associated with the value of θ , four alternatives are also considered: $\theta \in \{1, 2, 4, 5\}$.

⁴ The model can also be viewed as a version of the perpetual youth setup by Yaari (1965) and Blanchard (1985), with a time-varying discount coefficient that compensates for the age-dependent survival probabilities. The standard constant surviving probability cannot be used because it generates unrealistic distribution of life span in the model, which considerably affects the model's prediction regarding the distribution of wealth. For example, assuming the life expectancy of 60 years (agents start as 21-year-olds) the model has almost 15% of the agents who lived more than 120 years and 2% reached at least 240 years of age.

⁵ Certainly, only one set of parameters $\hat{\mathbf{b}}_{t,i,b}$ or $\hat{\mathbf{b}}_{t,i,g}$ is updated in one period, depending on the aggregate state of the economy.

⁶ It is assumed that the agents are born at the end of a period; hence, $t - s(i) = 1$ for a new-born agent.

Table 5
Results for the learning-from-experience model

	$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$	$\theta = 5$	RE
Characteristics of the wealth distribution						
\bar{K}	10.98	10.99	10.98	10.97	10.97	10.96
$sd(K)$	0.286	0.265	0.251	0.279	0.248	0.283
Gini	0.549	0.588	0.583	0.507	0.512	0.544
top 1 [%]	5.4	5.5	5.8	5.0	4.8	5.8
top 10 [%]	34.2	35.3	35.8	31.4	31.0	34.6
neg. wealth [%]	11.1	16.6	15.2	8.3	10.3	9.5
Characteristics of the expectations' heterogeneity						
$\hat{\psi}_1$	59.2	42.5	26.1	26.8	19.7	—
$\hat{\psi}_2$	7	9	3	4	1	—
$\hat{\Delta}_1$	0.001	0.133	0.161	0.153	0.278	—
$\hat{\Delta}_2$	<0.001	0.006	0.008	0.028	0.027	—
$sd(R_{impl})$ [p.p.]	<0.01	0.32	0.38	0.38	0.57	—
pf [%]	<0.01	1.5	2.2	1.6	3.5	—

The results are obtained from a simulation with 35,000 agents and 29,000 periods; the first 5000 periods are treated as a burn-in sample; “ $sd(K)$ ” denotes a standard deviation of the time series of aggregate capital; “top 1” and “top 10” represent average shares of total wealth held by the wealthiest 1% and 10% of agents, respectively; “neg. wealth” denotes average fraction of agents with negative wealth; the normalized Gini coefficient is calculated according to Chen et al. (1982) and Berrebi and Silber (1985) formula. $\hat{\psi}_1$, $\hat{\psi}_2$ – the mean and median, respectively, length of spells when the implied log aggregate capital defined as $0.5 \cdot (\log \bar{K}_{impl,b} + \log \bar{K}_{impl,g})$ stays continuously above or below the RE level; $\hat{\Delta}_1$, $\hat{\Delta}_2$ – the median cross-sectional standard deviation and interdecile range, respectively, of the implied log aggregate capital; “ $sd(R_{impl})$ ” – the median cross-sectional standard deviation of the implied expected interest rate expressed in percentage points; “pf” – the average fraction of cases per period when the projection facility is used.

4.1.3. A projection facility and a solution method

In order to exclude the estimates that generate the explosive dynamics of aggregate capital, a projection facility is introduced. The revised estimates are appropriately adjusted when it appears that they leave the stability region. The details of the facility are discussed in Appendix E. As shown below, the projection facility is rarely invoked, almost exclusively for the youngest agents, and is not likely to affect the results.

The learning scheme implies that the agents' expectations about the aggregate capital dynamics change in every period, which results in fluctuations in the decision rules. It is computationally infeasible to calculate the exact decision rules associated with all the sets of estimates $\hat{b}_{t,i,j}$ in a long simulation. Instead, the exact decision rules are calculated for a grid that is defined on the set of admissible values of \hat{b}_b and \hat{b}_g . Owing to a near-collinearity of the estimates \hat{b}_{0j} and \hat{b}_{1j} , it is possible to employ a simple version of the stochastic simulation approach to construct a sparse, adaptive grid that facilitates the computations. The grid is used for interpolating the decision rules for the simulated estimates. The detailed description of the computational procedure is provided in Appendix D.

4.2. Results

4.2.1. Wealth inequality

In the top panel of Table 5, several moments of the wealth distribution are reported. The expectations' heterogeneity does not affect the mean level of aggregate wealth, but significantly influences the wealth inequality. For the baseline calibration of the learning process, $\theta = 3$, the Gini coefficient rises from 0.544 to 0.583 and the wealth share of the wealthiest decile increases from 34.6% to 35.8%. The wealth share of the top 1% remains unaffected.

The rise in the Gini coefficient is also observed in the case of $\theta = 2$, but the effect for the wealth share of the wealthiest decile is smaller and even negative for the top percentile. Conversely, if $\theta = 4$ and $\theta = 5$, then the wealth concentration is lower compared to the RE model. The Gini coefficients slightly exceed 0.5 and the wealth shares of the top 1% and 10% drop to about 5% and 31%, respectively. In the case of $\theta = 1$, the results virtually coincide with those for the RE model.

4.2.2. Characteristics of the expectations' heterogeneity

The bottom panel of Table 5 characterizes dynamics of the agents' expectations in the studied model. The parameters $\hat{\psi}_1$ and $\hat{\psi}_2$ describe the length of spells when the implied aggregate capital defined as $0.5 \cdot (\log \bar{K}_{impl,b} + \log \bar{K}_{impl,g})$ stays continuously above or below the RE level. They can be treated as the empirical counterparts of the parameter ψ considered in Section 3. If $\theta = 3$, then the spells last about 26 quarters on average ($\hat{\psi}_1$), but the median ($\hat{\psi}_2$) equals 3 quarters only.

Generally, the expectations switch much faster as the gain parameter rises. If $\theta = 1$, then the mean time to the next transition reaches almost 60 quarters, whereas it is lower than 20 in the case of $\theta = 5$. For the median time, some non-monotonicity in this relationship can be observed; this suggests that the distribution of the spell length for the different values of θ varies not only with respect to the location but also with respect to the shape. This feature is documented in Fig. 4. It also depicts the extreme skewness of the analyzed distributions.

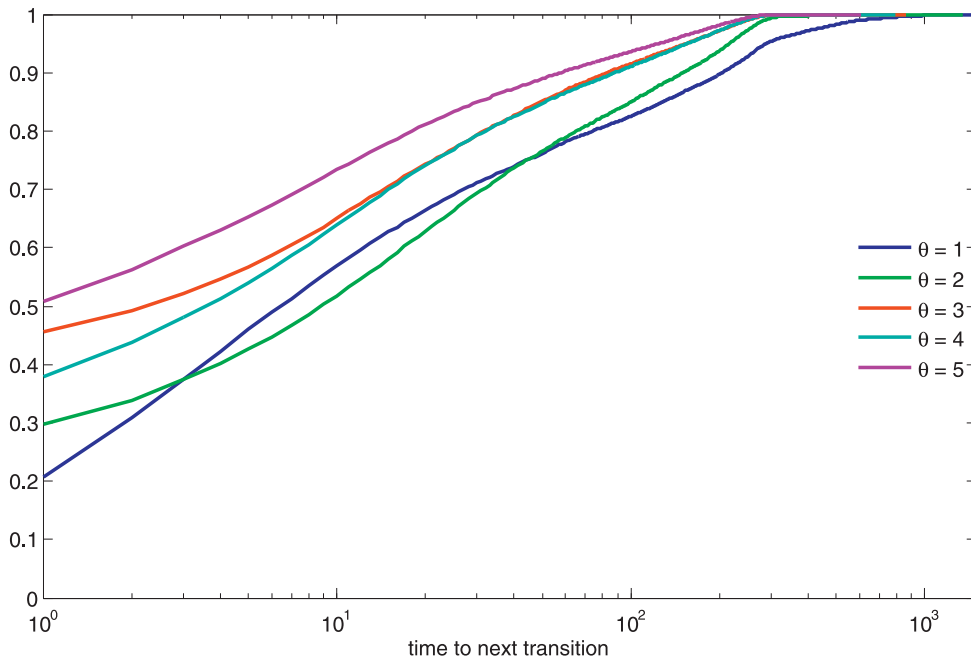


Fig. 4. Cumulative distributions of the time to next transition through the RE-implied aggregate capital stock level.

The level of the expectations' dispersion is described by the parameters $\hat{\Delta}_1$ and $\hat{\Delta}_2$ that can be considered as the empirical counterparts of the parameter Δ from the previous section. Apart from the case of $\theta = 1$, wherein the heterogeneity is very small, the cross-sectional standard deviation ($\hat{\Delta}_1$) of expectations is relatively high, but the interdecile range ($\hat{\Delta}_2$) is much lower. It means that the expectations of the vast majority of agents are close to each other, but the predictions are very far from the consensus for some agents. The dispersion is also positively correlated with the gain parameter.

Additionally, Table 5 reports the cross-sectional standard deviation of the implied expected interest rate. If $\theta > 1$, then the measure would vary between 0.3 p.p. and 0.6 p.p. This seems to be a rather low level of heterogeneity compared to the observed dispersion of inflation expectations. This can be explained by the fact that the agents could forecast the mean level of the interest rate quite accurately just a few periods after entering the labor market because of the low volatility of the interest rate generated by the model (see Table 3).

In Fig. 5, the negative correlation between the spell length and the average expectation bias for the spell is documented. The spells, wherein the mean implied aggregate capital significantly deviates from the RE level, hardly last longer than 50 periods; however, the time to transition for spells with the mean implied aggregate capital close to the RE level can exceed 300 periods.

4.2.3. Expectations dynamics

The properties of the expectations' heterogeneity analyzed above can be easily rationalized by studying the dynamics of the expectations of a single family shown in Fig. 6. The large jumps in the expectations observed for $\theta = 3$ and $\theta = 5$ are characteristic of the youngest agents. They tend to make large forecast errors because they do not accumulate any experience in the first few periods of their labor market career. The error overshoots in the opposite direction when these agents attempt to correct them in the subsequent periods. As the agents become older and use more observations, their expectations move toward the RE level, and the expectations' volatility rapidly decreases. Setting $\theta = 1$ implies that the agents use the standard least squares learning algorithm, and there are very small fluctuations in expectations.

4.3. Discussion

The observed changes in the wealth inequality can be explained taking into account the results obtained from the general model of heterogeneous expectations considered in Section 3. If $\theta = 1$, then the expectations change slowly and rarely cross the RE-implied capital stock level. However, the dispersion of the expectations is far too small to affect the distribution of wealth. When the gain coefficient rises ($\theta = 2$ or $\theta = 3$), the heterogeneity of the expectations becomes larger and the wealth inequality increases, despite the more volatile expectations. However, a further increase in the gain coefficient ($\theta = 4$ or $\theta = 5$) increases the volatility considerably, which eventually leads to a reduction in wealth concentration.

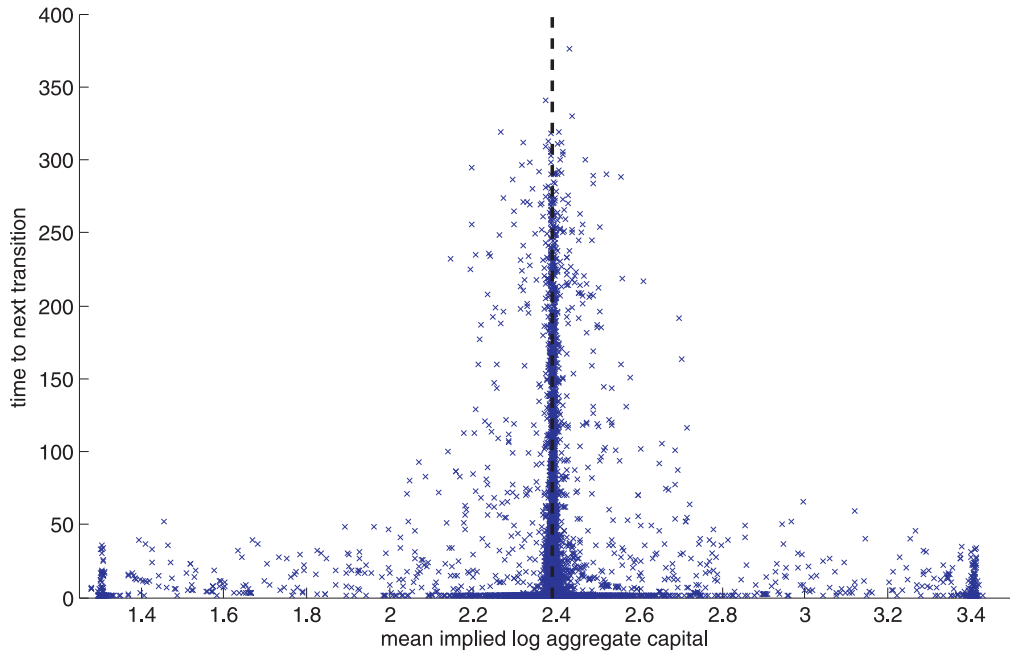


Fig. 5. Sample distribution of the time to next transition and the mean log implied aggregate capital for $\theta = 3$. On the horizontal axes, the mean levels of the implied log aggregate capital for the spells are shown. The dotted line indicates the RE level of the implied log aggregate capital.

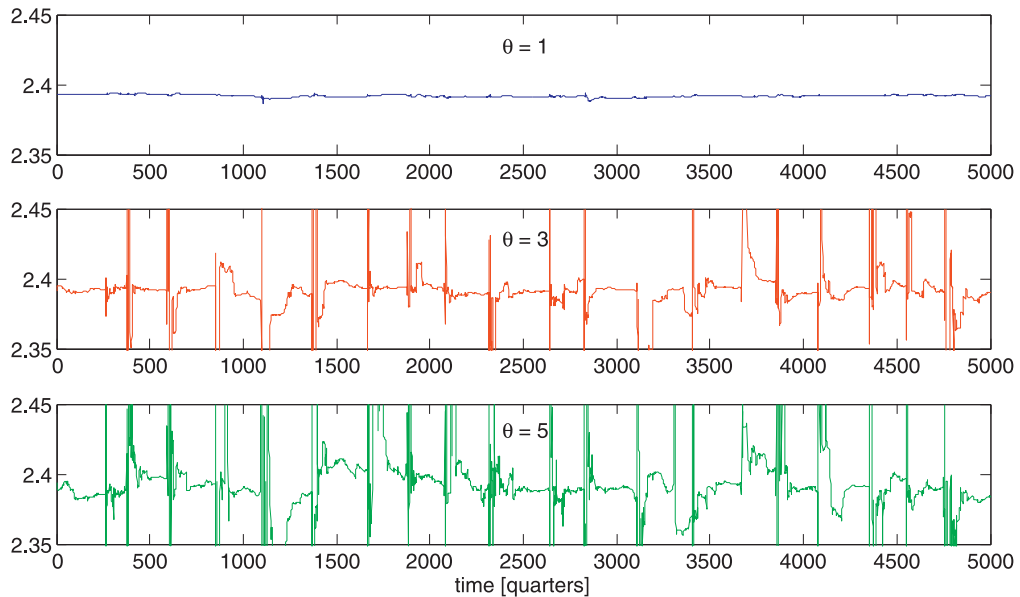


Fig. 6. Sample paths of the implied log aggregate capital under the learning-from-experience model

4.4. Robustness check

In this subsection, the sensitivity of the baseline results to the changes in the key parameters of the model is checked. The results of the exercise are presented in Table 6. The following three cases are studied: the baseline calibration used by Krusell and Smith (1998) with no unemployment benefits and zero borrowing constraint ($\mu = 0, \underline{k} = 0$), the higher risk aversion coefficient ($\gamma = 5$), and the larger aggregate shocks ($Z_b = 0.98, Z_g = 1.02$).

In the first case, wealth inequality in the learning-from-experience model rises compared to the RE version. In relative terms, the increase is even higher than in the baseline case presented above. A significant rise in wealth inequality is also

Table 6

Results for the alternative parametrizations of the model

	Krusell-Smith $\mu = 0, \bar{k} = 0$		Higher risk aversion $\gamma = 5$		Larger shocks $Z_b = 0.98, Z_g = 1.02$	
	HE	RE	HE	RE	HE	RE
Characteristics of the wealth distribution						
\bar{K}	11.03	11.03	11.05	11.06	10.99	10.98
$sd(K)$	0.247	0.275	0.423	0.469	0.341	0.375
Gini	0.286	0.240	0.344	0.388	0.596	0.512
top 1 [%]	3.4	2.6	2.7	3.4	5.9	5.1
top 10 [%]	22.1	19.1	21.2	24.3	36.5	31.9
neg. wealth [%]	0	0	3.3	3.4	16.7	8.3
Characteristics of the expectations' heterogeneity						
$\hat{\psi}_1$	27.4	—	35.9	—	28.2	—
$\hat{\psi}_2$	4	—	8	—	3	—
$\hat{\Delta}_1$	0.156	—	0.208	—	0.212	—
$\hat{\Delta}_2$	0.009	—	0.032	—	0.012	—
$sd(R_{impl})$ [p.p.]	0.37	—	0.48	—	0.53	—
pf [%]	2.1	—	2.5	—	2.2	—

The results are obtained from a simulation with 35,000 agents and 29,000 periods; the first 5000 periods are treated as a burn-in sample; " $sd(K)$ " denotes a standard deviation of the time series of aggregate capital; "top 1" and "top 10" represent average shares of total wealth held by the wealthiest 1% and 10% of agents, respectively; "neg. wealth" denotes average fraction of agents with negative wealth; the normalized Gini coefficient is calculated according to Chen et al. (1982) and Berrebi and Silber (1985) formula. $\hat{\psi}_1, \hat{\psi}_2$ – the mean and median, respectively, length of spells when the implied log aggregate capital defined as $0.5 \cdot (\log \bar{K}_{impl,b} + \log \bar{K}_{impl,g})$ stays continuously above or below the RE level; $\hat{\Delta}_1, \hat{\Delta}_2$ – the median cross-sectional standard deviation and interdecile range, respectively, of the implied log aggregate capital; " $sd(R_{impl})$ " – the median cross-sectional standard deviation of the implied expected interest rate expressed in percentage points; "pf" – the average fraction of cases per period when the projection facility is used. In all the heterogeneous expectations (HE) cases, $\theta = 3$ is assumed.

observed in the case of the larger shocks. For example, the Gini coefficient increases from 0.512 to 0.596 and the wealth share of the wealthiest percentile from 5.1% to 5.9%.

Conversely, the learning-from-experience mechanism lowers the wealth inequality in the case of higher risk aversion. For all the considered measures, the relative drop exceeds 10%, despite the fact of the expectations' heterogeneity in this variant being relatively large and persistent compared to the other studied calibrations. However, under the higher risk aversion, this is not enough to increase the concentration of wealth.

5. Concluding remarks

This study examines the impact of expectations' heterogeneity on wealth inequality. It is documented that this critically depends on the size and persistence of the cross-sectional differences between the agents' expectations. Although small, long-lasting differences in expectations can result in substantial wealth inequality. Conversely, wealth concentration can be reduced if expectations adjust fast. Both effects are observed in the empirically sound learning-from-experience model, according to which the expectations of young agents, who have little or even no experience, are highly volatile.

The study also shows that the impact of the expectations' heterogeneity on wealth inequality is sensitive to the parametrization of the macroeconomic and expectational parts of the model. Therefore, it would be useful to examine the implications regarding the level and persistence of dispersion of forecasts in other heterogeneous expectations models considered in the literature. It is quite likely that they would give very different results in this area, which would also mean different levels of wealth heterogeneity. However, this topic is left for further research.

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Supplementary materials

Supplementary material associated with this article, namely, the appendices, can be found, in the online version, at [doi:10.1016/j.jmacro.2017.06.009](https://doi.org/10.1016/j.jmacro.2017.06.009).

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