

# Pension reforms and welfare gains from eliminating business cycle in Poland

Jan Acedański<sup>1</sup>

**Abstract.** In the paper we study consequences of the changes in the pension system in Poland for the welfare gains from eliminating business cycle fluctuations. Using an overlapping generation model with aggregate fluctuations and exogenous idiosyncratic labour market risk we quantitatively show that raising the retirement age significantly reduces, whereas lowering the replacement rate from the public part of the pension system increases the business cycle costs. The impact of these changes is different as far as consumers with different skill level and wealth are concerned.

**Keywords:** welfare, business cycle, pension system, overlapping generations

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**AMS classification:** 90C15

## 1 Introduction

The recent changes in the demographic structure in both developed and developing countries have brought serious consequences for their pension systems. Decreasing ratio of active population to retirees forces the policy makers to implement deep reforms of the labour markets and the pension systems. As a result in most European countries the retirement age has been increased whereas the replacement rates of public pensions are expected to drop significantly.

This is also the case in Poland. Before 2012 year the official retirement age in Poland was 65 years for men and 60 years for women. However in many cases due to privileges for some professions it was even lower so that in 2012 on average people exited the labour market at about the age of 60 [9]. Since 2012 year the retirement age has been gradually being increased to 67 years for both sexes. Similarly currently the gross replacement rate in Poland is estimated at about 50% [6], whereas simulations show that it will be impossible to keep it constant and it is expected to fall significantly to about 30% [8].

Since the pension provides a relatively stable, usually long-lasting income, it offers a good protection against the business cycle fluctuations for risk-averse consumers. As the retirement age increases and the public pensions drop down the protection becomes weaker. This is because both income from work and interest from individual capital stock that have to replace the lower pensions, are more volatile at the business cycle frequencies compared to the pensions. However on the other hand the discussed changes allow the government to significantly lower tax rates. Then consumers can accumulate more wealth to hedge against the business cycle fluctuations. As a result the effect of the reforms for the labour market cyclical variation is qualitatively uncertain.

In the paper we examine quantitatively to what extent the discussed consequences of the pension system reforms in Poland affect the costs of business cycle fluctuations for people of different age and skill level. To achieve this goal we build a general equilibrium overlapping generation model with aggregate fluctuations and idiosyncratic labour market risk. The parameters of the model are calibrated to match basic characteristics of the labour market and the pension system in Poland before and after the reforms as well as some business cycle stylized facts. To assess the potential welfare gains from eliminating the business cycle fluctuations we employ the approach proposed by Lucas [4]. We study two versions of the model: with and without the aggregate risk. In the former case the amount of the idiosyncratic labour market risk depends on the aggregate state of the economy, whereas in the latter case it is constant. Then we study a transition path from the economy without the aggregate risk to the economy with business

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<sup>1</sup>University of Economics in Katowice, Department of Statistical and Mathematical Methods in Economics, ul. Bogucicka 14, 40-226 Katowice, Poland, jan.acedanski@ue.katowice.pl

cycle fluctuations. The welfare costs are defined as a percentage increase in individuals' consumption during the transition path needed to achieve the same lifetime utility from consumption as in the model without the aggregate risk. It should be stressed that the model does not take into account the changes in the demographic structure of the population.

The rest of the paper is organized as follow. In the next section we present the model. Then we discussed the model's calibration details. The fourth section contains the description of computational algorithms. Finally we show the results of the simulations and conclude the paper.

## 2 The model

We use a general equilibrium overlapping generation model with aggregate productivity shock and idiosyncratic risk following Storesletten, Telmer and Yaron [7]. However contrary to the cited work we do not focus on modelling exogenous individual income processes but rather assume that agents face exogenous labour market risk that is calibrated to match Polish data. Moreover we have a slightly different asset structure of the economy and allow for stochastic lifetime. We also assume exogenous labour supply as we exclude labour from the utility function. Below we describe the model with aggregate fluctuations. In the calibration section we highlight the differences between the model with and without the aggregate risk.

### 2.1 Description of the economy

The economy is populated by the continuum of finitely lived agents that differ in terms of age  $a$ , skill level  $s$ , employment status  $\epsilon$  and wealth  $k$ . For simplicity we omit the time subscripts. The agents start at the age of 20, work for 40 years, then retire and live at most up to 100 years. Young agents either work or are unemployed. If employed, they supply  $\xi_{s,a}l$  effective units of labour and get pre-tax income  $\xi_{s,a}lW$ , where  $W$  stands for the aggregate wage. Unemployed agents collect the unemployment benefits that are proportional to mean wage in the economy  $\theta_u\bar{\xi}lW$ , where  $\theta_u$  is the unemployment replacement rate and  $\bar{\xi}$  denotes mean labour efficiency across agents. Retirees receive pensions that are proportional to their last income from work  $\theta_r\xi_{s,59}lW$ , where  $\theta_r$  represents the pension replacement rate. Moreover all agents collect interest from their wealth according the the aggregate interest rate  $R$ . We assume that agents who die leave their capital for new-born agents that replace them, but do not take into account leaving the bequest when facing their consumption-investment problems. This assumption helps to keep the model closer to the data without making the agents decision horizon infinite.

The production sector consists of one representative firm that hires capital and labour from agents producing a single consumption good according to standard Cobb-Douglas technology:  $Y = ZK^\alpha L^{1-\alpha}$ , where  $Z$  is stochastic aggregate productivity shock,  $K = \int k_i di$  is aggregate capital stock and  $L = l \int \xi_{s,a} \mathbf{I}(a_i \leq 59, \epsilon_i = e) di$  is aggregate effective labour supply. Aggregate wage and interest rate are given by:  $W = (1 - \alpha)ZK^\alpha L^{-\alpha}$  and  $R = \alpha ZK^{\alpha-1} L^{1-\alpha}$ .

There is also government in the model who imposes taxes on the income from work to finance the unemployment benefits and the pensions. The tax rate is set so the government budget is balanced every period.

### 2.2 Agents' decision problem

Every period an agent  $i$  of age  $a$  faces a following consumption-saving problem. Given her capital stock  $k$ , aggregate state of the economy  $Z_t$  and individual income  $d$  she has to decide how much of her current wealth to consume and how much to save to maximise her lifetime utility. The problem can be written in the form of Bellman equation:

$$V(k, a, \epsilon, s, K, Z) = \max_{c, k'} \{U(c) + \beta q_{a, a+1} \mathbb{E}[V(k', a + 1, \epsilon', s, K', Z') | \epsilon, K, Z]\} \quad \text{s.t.} \quad (1)$$

$$k' = (1 - \delta + R)k + d - c, \quad (2)$$

$$d = \mathbf{I}(a \leq 59, \epsilon = e)(1 - \tau)\xi_{s,a}lW + \mathbf{I}(a \leq 59, \epsilon = u)\theta_u\bar{\xi}lW + \mathbf{I}(a > 59)\theta_r\xi_{s,59}lW, \quad k' \geq 0, \quad (3)$$

where  $V$  — value function,  $\beta$  — discount factor,  $q_{a,a+1}$  — survival probability from age  $a$  to  $a + 1$ ,  $\delta$  — capital depreciation,  $c$  — individual consumption,  $\tau$  — tax rate,  $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$  — one-period CRRA utility function,  $\gamma$  — risk aversion parameter. For the notation transparency we also omitted individual subscripts and denoted next period with the prime. In (3) we impose borrowing constraints as the capital holding can not be negative.

The Euler equation describing an agent's decision rule regarding next period's capital when the borrowing constraint is not binding has the following form:

$$k' = (1 - \delta + R)k + d - \left[ \beta q_{a,a+1} \mathbb{E} \left( \frac{1 - \delta + R'}{[(1 - \delta + R')k' + d' - k'']^\gamma} \right) \right]^{-\frac{1}{\gamma}}. \quad (4)$$

### 2.3 Stochastic structure of the economy

There are three stochastic shocks in the model. The aggregate productivity shock  $Z$  is represented by a two state Markov chain with transition matrix  $P_Z$ . The states of  $Z = \{Z_b, Z_g\}$  represent recession and expansion periods in the economy. The individual employment shock  $\epsilon$  is also modelled as a two state Markov chain with transition matrix  $P_\epsilon(Z, Z', s, a)$  that depends on the current and future state of the economy as well as agent's skill level and age. Finally there is also a stochastic lifetime. For every generation  $a$  there is a fraction  $1 - q_{a,a+1}$  of agents who die. It should also be noted that we do not allow agents to change their skill level.

## 3 Calibration

A period in the model corresponds to one year. Baseline calibration is presented in table 1. Share of capital in the production function  $\alpha$  is set at 0.36 which is a standard choice for models of this class. The values of the capital depreciation rate  $\delta = 0.032$  and discount rate  $\beta = 0.98$  are calibrated jointly to match the interest rate of about 5% and average share of investment in output that is about 20% in Poland. In our baseline calibration we assume coefficient of relative risk aversion  $\gamma = 5$ . The compensation rates for agents who does not work equal  $\theta_u = 0.15$  and  $\theta_r = 0.5$ , which roughly match the average values observed in Poland. As far as the skill level is concerned we have three group of agents in the model: low-, medium- and high-skilled which broadly correspond to people with primary, secondary and tertiary education level. Their average shares in Polish population equal 15%, 70% and 15% respectively.

Description	Parameter	Value
share of capital in the production function	$\alpha$	0.36
capital depreciation rate	$\delta$	0.032
discount rate	$\beta$	0.98
risk aversion	$\gamma$	5
unemployment benefit rate	$\theta_u$	0.15
pension replacement rate	$\theta_r$	0.5
share of low-, medium- and high-skilled workers	$\omega$	{0.15, 0.7, 0.15}
individual labour supply	$l$	1

**Table 1** Baseline calibration of the model

The individual efficiency factors  $\xi$  shown in table 2 are calibrated to match observed differences in wages according to the Structure of Earnings Survey conducted by Eurostat. The data show that earnings of low-skilled (primary-educated) workers are about 20% lower than that for medium-skilled (secondary-educated), whereas for high-skilled (tertiary-educated) they are about 65% higher. Similarly it is documented that young people (20-29 years) earn about 20% lower wages than the rest. The differences for other age groups are small so we neglect them.

To calibrate Markov chain for the aggregate productivity we calculate log-deviations from HP-filtered trend of yearly data on Polish GDP covering period 1996-2013. This implies a symmetric business cycle where each phase lasts 3.75 years on average and values of the productivity shock  $Z_b = 0.99$  during

	Skill level			Age		
	Low	Medium	High	20-29	30-59	
$\xi_s$	0.8	1	1.65	$\xi_a$	0.8	1

**Table 2** Calibration of the individual efficiency factors

recessions and  $Z_g = 1.01$  for expansions. The transition probabilities  $P_\epsilon$  are calibrated to match the average level  $\bar{u}$  and duration  $u_l$  of unemployment in Poland during booms and recession for workers of different skill level and age based on the data from the Labour Force Survey. Since the data on mean level of unemployment are grouped into 5 year bins, the values for each age were interpolated. Due to scarcity of data on the unemployment length we assume that it is equal for all education levels and distinguish only two age groups: 20-39 and 40-59. We calculate the transition probabilities in the following way. The probability of staying unemployed:  $p_{uu}(a, Z) = 1 - u_l^{-1}(a, Z)$ . The probability of losing a job:  $p_{eu}(a, s, Z) = [\bar{u}(a+1, s, Z') - \bar{u}(a, s, Z) \cdot (1 - p_{uu}(a, Z))] / [1 - \bar{u}(a, s, Z)]$ , where  $\bar{u}(a, s, Z)$  denotes mean unemployment rate for worker of age  $a$ , skill level  $s$  when the state of the economy is  $Z$ . Finally the survival rates  $q(a, a+1)$  are taken from Polish unisex life tables from 2012 year.

For the model without the aggregate risk we set  $Z = 1$ . Moreover to calibrate the labour market transition probabilities we take mean level and duration of unemployment for all periods regardless of the business cycle phase. However we still take into account the differences across age and skill level.

#### 4 Computational algorithm

Solving the model with the aggregate risk on the transition path we follow the general approximate aggregation algorithm proposed by Krusell and Smith [3] adapted to the overlapping generation models by Storesletten, Telmer and Yaron [7]. We assume that consumers are boundedly rational in the sense that they do not use the whole wealth distribution to predict next period aggregate wage and interest rate, which is an infinite dimensional object, but rather take into account only mean level of the aggregate capital  $K$ . They use simple autoregressive models to predict the next period stock of the aggregate capital:

$$K' = b_{0b} + b_{1b}K \text{ if } Z = Z_b, \quad K' = b_{0g} + b_{1g}K \text{ if } Z = Z_g, \quad b = [b_{0b}, b_{1b}, b_{0g}, b_{1g}]. \quad (5)$$

Krusell and Smith [3] proposed an iterative algorithm to find the coefficients  $b$  that are almost perfectly consistent with the actual dynamics of the aggregate capital in the model. It consists of the following steps:

1. Set initial values of the coefficients  $b^0$ .
2. For the given  $b^j$  find the individual's decision rules  $c = c(k, a, \epsilon, s, K, Z, b^j)$ ,  $k' = k'(k, a, \epsilon, s, K, Z, b^j)$  that solve the individual's consumption-saving problem (1)-(3).
3. Given the decision rules simulate the model for  $T$  periods starting from no aggregate risk stationary wealth distribution and compute a time path for the aggregate capital  $K$ .
4. Estimate the new autoregressive coefficients  $b_n^j$  using ordinary least squares.
5. If  $|b^{j+1} - b^j| < \nu_{tol}$  then stop, otherwise update the coefficients  $b^j$  and return to step 2.

As far as the second step is considered we use a backward iteration method. Since agents do not derive utility from leaving bequests, the optimal decisions for the last generation are  $k' = 0$  and  $c = k + d$ . Then we use this solution to find the decision of earlier generation. Here we employ the Euler equation (4) iteration approach proposed by Maliar, Maliar and Valli [5]. We discretize the state space for the individual and aggregate capital and look for the optimal next period's capital by iterating the Euler equation (4). More precisely with some initial decision rule  $k'$  and coefficients  $b^j$  we calculate the r.h.s. of (4) on the predefined grid and use it to update the decision rule. We iterate until convergence. To calculate the expectation term the transition probabilities are used as well as the decision rule of the next generation (to obtain  $k''$  term).

In the third step we simulate the model. Here we do not perform full Monte Carlo simulations with some predefined number of agents. Instead we follow Heer and Maussner [2, p. 541–545] and simulate only aggregate productivity shock. Then for every period we analytically compute individual capital density functions for each skill level and age by taking advantage of the fact that with discretized individual capital level its dynamics is described by a Markov chain with transition probabilities depending on the current and the future state of the economy.

Finally it should be noted that for updating the parameters  $b$  as well as the individual decision rules  $k'$  we use the following updating rules:  $b^{j+1} = \phi_b b^j + (1 - \phi_b) b_n^j$  and  $k'^{j+1} = \phi_k k'^j + (1 - \phi_k) k_n'^j$ .

In the model without the aggregate risk the aggregate capital is constant and so are the interest rate and the aggregate wage. As a result the model solution is easier. However rational agents still have to know the aggregate level of capital to accurately forecast interest rate and wage. Therefore we employ a similar solution strategy as in the model with the aggregate fluctuations assuming trivial capital forecasting rule  $K = b$ . And in such setup the decision rules obviously no longer depend on  $K$ .

For the individual capital holdings the decision rule are approximated on the interval  $[0, 50]$  with 100 grid points. Following Maliar, Maliar and Valli [5] we use a polynomially spaced grid of 7<sup>th</sup> order that places more points near the borrowing constraint where the decision rule is far from linear. For the aggregate capital we use uniformly spaced 10-points grid which support is tuned for every parametrization we study. We simulate the law of motion for the aggregate capital for  $T = 200$  periods. In this step we use equispaced interval for the individual capital stock with 200 grid points. The updating parameters equal:  $\phi_b = 0.4$  and  $\phi_k = 0.4$ .

## 5 Results

The welfare gain from eliminating the business cycle fluctuations for an agent is defined as a percentage increase in individuals' consumption on the transition path needed to achieve the same lifetime utility from consumption as in the model without the aggregate risk. To compute the cost we simulate 100 consumption distribution paths in the two economies. Then for every simulated period we calculate the lifetime discounted utilities from the consumption for agents with median consumption who die at age 100 in that period. For every two lifetime utilities we solve for a constant consumption increase rate on the transition path, that makes the two lifetime utilities equal. Below we report mean values over 100 simulations. To assess the consequences of the pension reforms for the business cycle costs we consider economies in which the pension replacement rate drops to 25% as well as the case when the retirement age is increased from 60 to 67 years.

Model	All agents	Skill level			Wealth percentile	
	(median wealth)	Low	Medium	High	5	95
Baseline	2.03 (1.26)	1.82 (1.01)	2.19 (1.44)	2.38 (1.65)	-1.48 (-1.64)	3.14 (2.29)
$\theta_r = 0.25$	2.46 (1.69)	2.25 (1.48)	2.60 (1.83)	2.93 (2.11)	-0.38 (-0.72)	3.31 (2.43)
$RA = 67$	1.12 (0.59)	0.90 (0.37)	1.28 (0.77)	1.27 (0.79)	-1.33 (-1.45)	2.34 (1.65)
$\theta_r = 0.25,$ $RA = 67$	1.51 (0.97)	1.37 (0.83)	1.61 (1.07)	1.65 (1.14)	-1.10 (-1.28)	1.14 (0.56)

Table 3 Maximum costs of business cycle (in percent of lifetime consumption) for median consumers (mean costs in parentheses)

The costs of business cycle under different pension system characteristics are presented in table 3. In every case we report two values: the maximum lifetime costs which is usually associated with an agent who is born at the beginning of the transition path and dies in 80<sup>th</sup> period, as well as the mean costs for agents who die at age 100 between 60 and 120 year of the transition period. The first column contains costs for all agents with median wealth level. Then we present also results for agents with different skill level as well as wealth.

In baseline version of the model maximum cost of the business cycle fluctuations is about 2% of lifetime consumption and on average it is 1.3%. It also increases with the consumers skill level. It is also worth mentioning that for the poorest consumers the cost is negative, which means that they are better

in the economy with fluctuations. This is because relatively large part of their income comes from the social security system that, due to higher capital accumulation, is more generous in the economy with the business cycle fluctuations.

With lower pension replacement rate the costs rises significantly to 2.5% and 1.7% respectively. In relative terms the reform would boost the costs by 25%. Again the increase is higher for high-skilled workers. Interestingly it affects especially the poorest people and is virtually harmless for the wealthiest consumers. Increasing the retirement age on the other hand halves the costs to 1.1% and 0.6% respectively regardless of the skill level. However contrary to the previous case the rise of the retirement age is beneficial mainly for the wealthiest consumers but do not matter for the poorest. Finally considering simultaneously the lower replacement rate and higher retirement age the maximum cost decreases to about 1.5% and mean cost to 1% of the lifetime consumption. The drop is highest for high-skilled and wealthiest consumers.

## 6 Conclusion

In the paper we quantitatively studied the consequences of the pension system reforms for the cost of the business cycle fluctuations in Poland. The results of our simulation study suggest that raising the retirement age significantly reduces, whereas lowering the replacement rate from the public part of the pension system increases the business cycle costs. Overall the two changes reduce the maximum cost from 2% to 1.5% of lifetime consumption for the median consumer. That means that the benefits from increasing individual capital accumulation possibilities under the new pension scheme outweigh the negative effects of less generous pensions. This is the case especially for high skilled and wealthiest consumers, although they still suffer the most from the business cycle fluctuations on labour market.

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