

Demographics, Retirement Age, and Real Interest Rates in Poland

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Abstract

Changes in the size and the age structure of a population have a great impact on an economy, especially on national savings and capital flows. Poland's population, although still relatively young when compared to other developed countries, is expected to experience accelerated ageing and decline in forthcoming decades. In this paper, we assess the effects of these processes for Polish economy. Using an open-economy OLG model with demographic shocks and a variable retirement age, we simulate dynamics of real interest rates, main macro aggregates as well as net foreign assets to GDP. We show that rapid ageing will reduce the interest rate gap between Poland and the developed countries by 1.3-2 p.p. We also document a strong positive relationship between interest rates and the retirement age and find that the decline in the interest rate in Poland is primarily driven by the surviving probability shock.

Keywords: demographic transition, ageing, old-age dependency ratio, real interest rates, economic growth

JEL Classification: E43, F41, J11, J26

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1 Introduction

Demographic processes, such as decline in fertility rates, ageing, and depopulation, are among the most conspicuous challenges facing modern societies and economies around the world, especially in developed countries. These processes will be especially visible in Poland which is a country expected to experience a dramatic pace of ageing even compared to other developed countries.

Demographics has a profound, long-term impact on macroeconomy. It affects *inter alia* national savings, capital-labor ratio, wage rates, fiscal position, and current account balance (Auerbach *et al.*, 1989). It is a key determinant of the long-term real interest rate called also the natural rate of interest. Demographics affects the interest rate through three channels, at least. Declining labor supply due to a decreasing population growth rates lowers marginal productivity of capital and thus the interest rate. Increasing expected retirement duration due to a longer life expectancy induces saving leading to a decline in the interest rate. Finally, in an open economy, decreasing world interest rate caused by population ageing in the global scale affects the domestic interest rate alike. The exact strength of influence depends on the dynamics of demographic processes as well as on implemented policies related to labor market, pension system, and the retirement age which can mitigate the demographic effects to some extent.

The aim of this paper is to investigate the effects of demographic processes on macroeconomic performance of Poland's economy. To accomplish this task, we develop an open-economy version of the overlapping-generations model used by Carvalho *et al.* (2016). This is a Blanchard-Yaari type of model with demographic shocks. We calibrate its parameters to match the key features of Poland's economy and projected demographic tendencies in Poland. We track responses of macroeconomic aggregates to these shocks and compare them with analogous predictions for the developed world. We also study sensitivity of the results to changes in the domestic and global effective retirement age.

Following the literature (McMillan, Baesel, 1990; Lenahan, 1996; Miles, 1999; Krüger, Ludwig, 2007; Ikeda, Saito, 2014; Aksoy *et al.*, 2016; Carvalho *et al.*, 2016; Gagnon *et al.*, 2016; Zhu, 2016; Kara, von Thadden, 2016; Eggertsson *et al.*, 2017; Sudo, Takizuka, 2018; Acedanski, Włodarczyk, 2018), we focus on the domestic real interest rate, but we also study its relation to the world interest rate. Similarly to previous works, we project a downward tendency for the real interest rate in Poland due to ageing. Apart from interest rates, and other frequently analyzed macro aggregates including GDP, consumption and investment, we also study the effects of demographic processes on international capital flows and simulate the ratio of net foreign assets to GDP. This approach was earlier adopted by Brooks (2003), Börsch-Supan *et al.* (2006), Attanasio *et al.* (2007), Krüger, Ludwig (2007), Ferrero (2010), Backus *et al.* (2014) and Lisack *et al.* (2017). For instance, Börsch-Supan *et al.* (2006) predict substantial capital flows from fast-ageing countries to the rest of the world that are likely to reverse when households start to decumulate saving.

In the open-economy setting capital mobility plays an important role (cf. Elmendorf, Sheiner, 2000; Guest, 2006). Most of the papers on macroeconomic effects of ageing analyze two extremes: a closed economy with capital perfectly immobile or a small open economy with the domestic interest rate equal to an exogenous world interest rate. Sometimes, in order to keep the model analytically tractable, the interest rate is held constant (Heijdra, Romp, 2009a,b). In our setup capital is not perfectly mobile and the domestic interest rate is endogenous. Similarly to Guest (2006) and Kudrna, Woodland (2011), we treat the domestic interest rate as a linear function of foreign debt measured by net foreign assets to GDP.

On the whole our work is closest to the paper by Bielecki *et al.* (2017) who also investigate effects of demographic processes on Poland's economy using an open-economy setting. However, they use a different analytical framework, concentrate on the perspective of monetary policy and analyze linkages between Poland's economy and the euro area. Furthermore, we are able to assume a variable retirement age and consider several alternative projection variants. Despite the reduction of the statutory retirement age in Poland in 2017 we assume a gradual increase in the effective retirement age. The impact of increasing the retirement age on welfare is discussed *inter alia* by Tyrowicz *et al.* (2016), Bielecki *et al.* (2016), and Vogel *et al.* (2017).

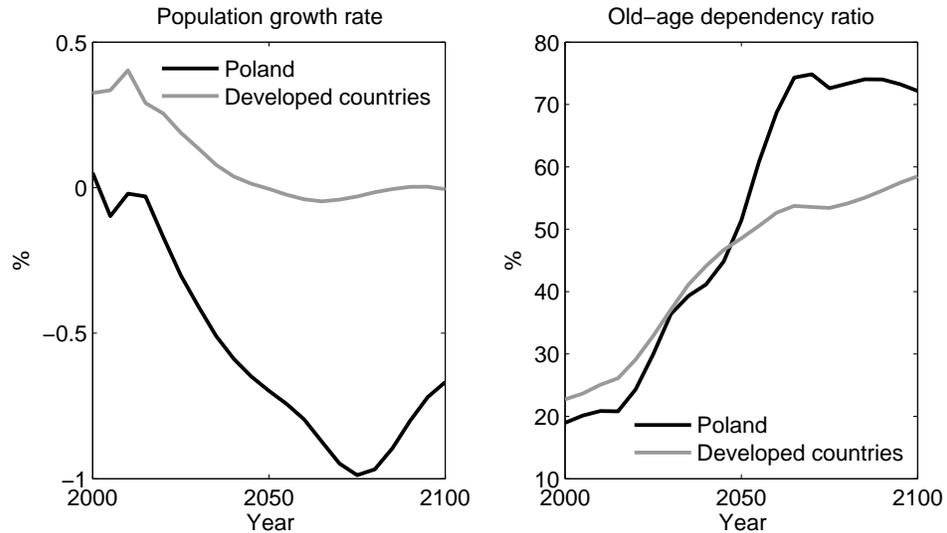
The remainder of this paper is organized as follows. Section 2 describes the current and projected demographic situation in Poland. Section 3 presents the model adopted for simulations. Section 4 explains the calibration procedure. Section 5 discusses baseline results which are supplemented with some additional results presented in Section 6. Final section concludes.

2 Demographic processes in Poland

Poland's population is still relatively young. According to UN estimates, the median age in Poland is lower than in developed countries treated as an aggregate (in the UN World Population Prospects the term "more developed regions" is used to encompass Europe, Northern America, Australia, New Zealand, and Japan), but this situation is likely to change already in 2025. Furthermore, total fertility rate is and is projected to remain one of the lowest worldwide. Thus in forthcoming decades Poland's population is expected to experience negative growth rates and accelerating process of ageing (see Figure 1).

Deep demographic depression in Poland has been determined mostly by two processes: post-war baby boomers entering the retirement age and economic transition in the late 1980s and early 1990s. Economic transition brought about many changes in the labor market and greater economic instability. These developments were coupled with increasing openness and easier emigration, as well as a diffusion of family patterns from other European countries. In Poland fertility is still strongly related to marriage, therefore lower frequency and postponement of marriage results in declining fertility

Figure 1: Projected demographic tendencies in Poland and developed countries as an aggregate. The old-age dependency ratio is defined as population aged 65 and above to population between 20 and 64 years old



Source: United Nations World Population Prospects 2017 database (medium scenario).

(Kotowska *et al.*, 2008). As a consequence, instead of the expected second baby boom echo effect during the 1997-2007 a first period of declining population occurred (Gawryszewski, 2005). The second period of declining population began in 2012. Altogether, the postwar period in Poland was characterized by following developments (Waligórska, Witkowski, 2016):

1. transition from dynamic population growth to slower growth rates and depopulation in recent years,
2. decrease in total fertility rate reaching sub-replacement fertility since 1989,
3. decrease in mortality rates during the transition period,
4. convergence of fertility behavior between urban and rural areas,
5. more intense emigration during the transition period and after opening of European labor markets.

These processes resulted in unfavorable changes in the age structure towards a decreasing share of pre-working age population and an increasing share of post-working age population. During forthcoming decades further changes in the age structure, accelerating ageing, systematic depopulation, and a dramatic decrease

of the working age population will take place (Waligórska, Witkowski, 2016). Nevertheless, the long run effects of ageing on economic variables like interest rates depend significantly on the assumptions about fiscal policy and the retirement age, as well as migration.

Poland is characterized by a relatively low exit age from the labor market as compared to other developed countries. Low level of engagement of older people in the labor market is typically associated with the entitlement to old-age pensions or pre-retirement pension schemes. The second reason is the state of health — in Poland the subjective assessment of health by people aged 65 and above is the lowest among OECD countries (OECD, 2013).

Furthermore, problems associated with ageing of the society may be the question of inappropriate policies and institutions (Bloom *et al.*, 2010), as well as — especially in the case of Poland — social expectations. After economic transition many people in Poland benefited from early retirement. This was supposed to lessen the pressure on the labor market. However, it created not only a greater pressure on the Social Insurance Institution, but also might have shaped expectations of early retirement among younger generations.

As far as the problem of migration is concerned, it is worth mentioning that the medium variant of World Population Prospects depicted in Figure 1 is based on following assumptions: medium fertility, normal mortality and normal international migration. For Poland this means negative net migration. Although lately the immigration to Poland has sharply increased and according to the Eurostat (2018) in 2017 Poland issued the highest number of first residence permits to non-EU citizens, these developments may not be persistent. Firstly, an increase in the number of permits issued was caused by favorable economic conditions in Poland. This is associated with business cycle frequency, while in this paper we analyze long-run trends. There is not much empirical evidence allowing to assume that in case of the majority of immigrants a temporary stay will turn into a permanent residence which could significantly alter our simulations. Secondly, a new immigration law aimed to fill the gaps in the market for skilled labor in Germany is expected to come into force soon. Therefore, even though increased immigration to Poland is not fully captured by projections underlying our computations, we hold on to them.

3 Model

We use a small open-economy version of the OLG model developed by Gertler (1999) and Carvalho *et al.* (2016) and recently employed by Acedański, Włodarczyk (2018). It allows for unexpected demographic shocks and a variable retirement age. There are three types of agents in the economy: households, firms and the government. The model accounts for three channels through which demographics affects interest rates in an open economy: declining labor supply due to decreasing population growth rates,

increasing expected retirement duration due to longer life expectancy and decreasing global interest rates caused by the global population ageing.

3.1 Households

The economy is inhabited by a continuum of households who are either workers (w) or retirees (r). The number of workers and retirees in period t is denoted by N_t^w and N_t^r , respectively. In period t , a worker remains in the labor force with probability ω_t and retires otherwise. The life length of a retiree is stochastic and the probability of surviving from period $t - 1$ to period t is denoted by γ_t . To facilitate aggregation, both probabilities, ω_t and γ_t , are constant across individuals and independent of their age. The mass of workers grows at rate n_t :

$$N_t^w = (1 + n_t)N_{t-1}^w. \quad (1)$$

The evolution of the number of retirees is given by:

$$N_t^r = (1 - \omega_t)N_{t-1}^w + \gamma_t N_{t-1}^r. \quad (2)$$

The demographic structure of the population is represented by the retirement-age dependency ratio defined as the ratio of retired population to the number of workers:

$$\psi_t \equiv \frac{N_t^r}{N_t^w} = \frac{1 - \omega_t}{1 + n_t} + \frac{\gamma_t}{1 + n_t} \psi_{t-1}. \quad (3)$$

This measure differs from the old-age dependency ratio presented in figure 1, because ψ_t is based on the actual status on the labor market, while the old-age dependency ratio refers simply to the age of individuals.

Following Carvalho *et al.* (2016), we assume that the retirement probability depends on the surviving probability. More specifically:

$$\omega_t = \omega_0 + \iota(\gamma_t/\gamma_0 - 1), \quad (4)$$

where ι governs sensitivity of ω_t to changes in γ_t and ω_0 and γ_0 denote initial values of ω_t and γ_t . The initial values, which are the starting point of the simulations, coincide with the model's steady state. This specification captures the increase in the retirement age implemented lately as a remedy to pension systems imbalances in many developed countries.

Households' preferences are described by a non-expected risk neutral utility function that takes the following recursive form:

$$V_t^z = \begin{cases} \{(C_t^w)^\sigma + \beta [\omega_{t+1} V_{t+1}^w + (1 - \omega_{t+1}) V_{t+1}^r]^\sigma\}^{1/\sigma} & \text{if } z = w, \\ \{(C_t^r)^\sigma + \beta \gamma_{t+1} (V_{t+1}^r)^\sigma\}^{1/\sigma} & \text{if } z = r, \end{cases} \quad (5)$$

where $z = \{w, r\}$, C_t^z denotes consumption of the group z and V_t^z is the value function. The expression $(1 - \sigma)$ is the inverse of elasticity of intertemporal substitution. The difference in the effective discount rates between workers (β) and retirees ($\beta\gamma_{t+1}$) reflects the death probability in the latter group. Retirees cannot return to the workers' group and their expected value part of the formula is simpler.

Retirees

Income of a retired household consists of a pension and interest on accumulated wealth. The wealth is stored in the form of physical capital K and one-period risk-free bonds B issued by the government. Additionally, agents have access to international financial market where they can borrow capital paying a premium $P(F)$ over the world interest rate R^* , where F denotes total foreign assets in the economy. It is assumed that at the beginning of each period retirees invest their total wealth into investment funds operating on a perfectly competitive market that pay back a premium over the market return equal to $1/\gamma_t$. This compensates for the probability of death. Wealth of a dead household is collected by the government and spent on public consumption. A slightly more realistic assumption that bequests are distributed equally across workers does not change the main results, as documented by Acedański, Włodarczyk (2018). In period t , the decision problem of a retiree born in period j and retired in period τ takes the following form:

$$\max_{C_t^r(j,\tau), K_t^r(j,\tau), B_t^r(j,\tau), F_t^r(j,\tau)} \{(C_t^r(j, \tau))^\sigma + \beta\gamma_{t+1}[V_{t+1}^r(j, \tau)]^\sigma\}^{1/\sigma}, \quad (6)$$

subject to:

$$\begin{aligned} C_t^r(j, \tau) + K_t^r(j, \tau) + B_t^r(j, \tau) - F_t^r(j, \tau) = \\ = \frac{(1 - \delta + R_t^K)K_{t-1}^r(j, \tau) + R_{t-1}B_{t-1}^r(j, \tau) - [R_{t-1}^* + P(F_{t-1})]F_{t-1}^r(j, \tau)}{\gamma_t} + E_t^r, \end{aligned} \quad (7)$$

where C_t^r is consumption, F_t^r — foreign assets, R_t^K and R_t denote the interest rates on capital and bonds, respectively, E_t^r stands for the lump-sum pension transfer, and δ is the capital depreciation rate.

The Euler equations for bonds, capital, and foreign assets imply that the interest rates on all assets are equal:

$$R_t = R_{t+1}^K + 1 - \delta = R_t^* + P(F_t). \quad (8)$$

From the first-order optimality conditions one can derive the consumption function for retirees:

$$C_t^r(j, \tau) = \xi_t^r \left[\frac{R_{t-1}A_{t-1}^r(j, \tau)}{\gamma_t} + S_t^r \right], \quad (9)$$

where $A_t^r(j, \tau) = K_t^r(j, \tau) + B_t^r(j, \tau) - F_t^r(j, \tau)$ denotes total assets of the household, S_t^r is the present discounted value of pension transfers:

$$S_t^r = E_t^r + \frac{S_{t+1}^r \gamma_{t+1}}{R_t} \quad (10)$$

and ξ_t^r denotes the marginal propensity to consume satisfying the following first-order difference equation:

$$\xi_{t+1}^r \left(\frac{R_t A_t^r(j, \tau)}{\gamma_{t+1}} + S_{t+1}^r \right) = (\beta R_t)^\sigma \xi_t^r \left(\frac{R_{t-1} A_{t-1}^r(j, \tau)}{\gamma_t} + S_t^r \right). \quad (11)$$

Workers

Workers enter the labor market with no assets and inelastically supply one unit of labor in each period. Their income consists of a wage and interest on assets. The decision problem of a worker born in period j expressed in terms of total assets: $A_t^w(j) \equiv K_t^w(j) + B_t^w(j) - F_t^w(j)$ can be written as:

$$\max_{C_t^w(j), A_t^w(j)} \{ (C_t^w(j))^\sigma + \beta [\omega_{t+1} V_{t+1}^w(j) + (1 - \omega_{t+1}) V_{t+1}^r(j, t + 1)]^\sigma \}^{\frac{1}{\sigma}}, \quad (12)$$

subject to:

$$C_t^w(j) + A_t^w(j) = R_{t-1} A_{t-1}^w(j) + W_t - T_t^w, \quad (13)$$

where W_t denotes the wage and T_t^w stands for lump-sum taxes.

From the first-order conditions, the consumption function of a worker takes the following form:

$$C_t^w(j) = \xi_t^w [R_{t-1} A_{t-1}^w(j) + H_t^w + S_t^w], \quad (14)$$

where ξ_t^w denotes the marginal propensity to consume of a worker, whereas H_t^w and S_t^w are the discounted present values of future wages net of taxes and pension benefits, respectively, defined as:

$$H_t^w \equiv W_t - T_t^w + \sum_{v=1}^{\infty} \frac{(W_{t+v} - T_{t+v}^w)}{\prod_{s=1}^v \Omega_{t+s} R_{t+s-1} / \omega_{t+s}} = W_t - T_t^w + \frac{\omega_{t+1} H_{t+1}^w}{\Omega_{t+1} R_t}, \quad (15)$$

$$\begin{aligned} S_t^w &\equiv \frac{(\Omega_{t+1} - \omega_{t+1}) S_{t+1}^r}{\Omega_{t+1} R_t} \sum_{v=1}^{\infty} \frac{(\Omega_{t+v+1} - \omega_{t+v+1}) S_{t+v+1}^r / \Omega_{t+v+1} R_{t+v}}{\prod_{s=1}^v (\Omega_{t+s} R_{t+s-1})} = \\ &= \frac{(\Omega_{t+1} - \omega_{t+1}) S_{t+1}^r}{\Omega_{t+1} R_t} + \frac{\omega_{t+1} S_{t+1}^w}{\Omega_{t+1} R_t} \end{aligned} \quad (16)$$

with:

$$\Omega_t \equiv \omega + (1 - \omega) \left(\frac{\xi_t^r}{\xi_t^w} \right)^{1/(1-\sigma)}. \quad (17)$$

The marginal propensity to consume evolves according to:

$$\frac{1}{\xi_t^w} = 1 + \beta^\sigma (\Omega_{t+1} R_t)^{\sigma-1} \frac{1}{\xi_{t+1}^w}. \quad (18)$$

Aggregation

The specification of the model described above allows for a straightforward aggregation of households' decision rules. The aggregates are denoted without the indices j and τ . The aggregate consumption of workers (C_t^w) and retirees (C_t^r) takes the same form as the consumption of an individual household of a given type:

$$C_t^w = \xi_t^w (R_{t-1}(1 - \lambda_{t-1})A_{t-1} + H_t + \tilde{S}_t), \quad (19)$$

$$C_t^r = \xi_t^r (R_{t-1}\lambda_{t-1}A_{t-1} + S_t), \quad (20)$$

where λ_t denotes the share of total assets $A_t = A_t^r + A_t^w$ held by retirees. This share evolves according to:

$$[\lambda_t - (1 - \omega_{t+1})]A_t = \omega_{t+1}[(1 - \xi_t^r)R_{t-1}\lambda_{t-1}A_{t-1} + E_t - \xi_t^r S_t]. \quad (21)$$

The dynamics of the aggregate non-financial wealth is described by the following equation:

$$H_t = W_t N_t - T_t + \frac{\omega_{t+1} H_{t+1}}{\Omega_{t+1} R_t}, \quad (22)$$

while \tilde{S}_t and S_t represent the aggregate discounted future pension benefits of workers and retirees and satisfy the following set of difference equations:

$$(1 + n_{t+1})\tilde{S}_t = \frac{(\Omega_{t+1} - \omega_{t+1})S_{t+1}/\psi_{t+1}}{\Omega_{t+1} R_t} + \frac{\omega_{t+1}\tilde{S}_{t+1}}{\Omega_{t+1} R_t}, \quad (23)$$

$$(1 + n_{t+1})S_t = (1 + n_{t+1})E_t + \frac{\psi_t S_{t+1} \gamma_{t+1}}{\psi_{t+1} R_t}. \quad (24)$$

Finally, the dynamics of the aggregate discounted future wages net of taxes H_t is governed by the following equation:

$$(1 + n_{t+1})H_t = (1 + n_{t+1})W_t N_t^w - (1 + n_{t+1})T_t + \frac{\omega_{t+1} H_{t+1}}{\Omega_{t+1} R_t}. \quad (25)$$

3.2 Production sector

The perfectly competitive production sector uses capital and labor to produce a homogeneous final good. The production technology is described by the standard Cobb-Douglas function with labor-augmenting technological progress:

$$Y_t = K_{t-1}^\alpha (X_t N_t^w)^{1-\alpha}, \quad (26)$$

where α is the capital share and X_t represents the technological progress with the constant growth rate x :

$$X_t = (1 + x)X_{t-1}. \quad (27)$$

Factor prices are equal to the marginal productivities of capital and labor:

$$R_t^K = \alpha \frac{Y_t}{K_{t-1}}, \quad (28)$$

$$W_t = (1 - \alpha) \frac{Y_t}{N_t^w}. \quad (29)$$

The final good is used for consumption and investment purposes.

3.3 Government

The public pension system in the model takes a form of the standard unfunded PAYG scheme. The government levies lump-sum taxes T_t to finance pensions E_t and public consumption that is proportional to output $G_t = gY_t$. It also issues one-period bonds. The amount of government debt is assumed to be constant and proportional to the output:

$$B_t = bY_t. \quad (30)$$

The government budget takes the following form:

$$T_t + B_t = R_{t-1}B_{t-1} + G_t + E_t. \quad (31)$$

3.4 World interest rate and the risk premium

As already mentioned in the introduction, the domestic interest rate R is endogenous due to imperfect capital mobility. We follow Guest (2006), Kudrna, Woodland (2011), and Rubaszek (2012), among others, and assume that the domestic interest rate is a linear function of foreign debt measured by net foreign assets to GDP:

$$R_t = R_t^* + P(F_t). \quad (32)$$

The dynamics of the world interest rate R^* is treated as exogenous, while the risk premium $P(F)$ is given by:

$$P(F_t) \equiv \phi \frac{F_t}{Y_t}, \quad (33)$$

where ϕ represents the economy's degree of openness. If $\phi = 0$ there is no premium at all and $R_t = R_t^*$. On the other hand, if $\phi \rightarrow \infty$ the economy is autarkic and $F_t \rightarrow 0$. The validity of the assumption stipulated by equation (32) may depend on the fact to what extent the country risk is transferred to the private sector (Casares, 2015).

3.5 Equilibrium

Given the demographic processes represented by n_t , ω_t , γ_t and ψ_t as well as the technological growth rate of the economy x and the world interest rate R^* , a competitive equilibrium for this economy is a sequence of quantities $\{C_t^r, C_t^w, A_t, \lambda_t, H_t, Y_t, K_t, I_t, B_t, T_t, F_t\}$, marginal propensities to consume $\{\xi_t^r, \xi_t^w, \Omega_t\}$, and prices $\{R_t, R_t^K, W_t\}$ such that:

1. Taking prices as given, households maximize lifetime utility subject to their budget constraints.
2. Firms maximize profits subject to their technology.
3. The government chooses taxes to satisfy its budget constraint.
4. The markets clear and the resource constraint is satisfied:

$$Y_t = C_t + I_t + G_t + [R_{t-1}^* + P(F_{t-1})]F_{t-1} - F_t, \quad (34)$$

where:

$$I_t = K_t - (1 - \delta)K_{t-1}. \quad (35)$$

To find the time-invariant solution of the model, stationary variables $s_t \equiv S_t/(X_t N_t^w)$ are introduced where necessary. Then, the steady state is calculated and the dynamics of the model around the steady state is studied.

In order to investigate the role of demographics in shaping the world interest rate, we use a closed version of the model presented above for developed countries.

3.6 Exogenous shocks

It should be noted that there is no aggregate uncertainty in the model presented so far and the equilibrium discussed above can be viewed as a perfect foresight one. In order to study the effects of changes in demographic characteristics, we consider a single unexpected shock for n_t and γ_t that generates a long-lasting hump-shaped response in these characteristics. To capture the impact of the decreasing world interest rate a similar shock for R_t^* is introduced. More specifically, we assume that these shocks are characterized by:

$$n_t = (1 + n_0) \exp(u_{nt} - v_{nt}) - 1, \quad (36)$$

$$\gamma_t = \gamma_0 \exp(u_{\gamma t} - v_{\gamma t}), \quad (37)$$

$$R_t^* = R_0^* \exp(u_{R^* t} - v_{R^* t}), \quad (38)$$

where n_0 , γ_0 , and R_0^* stand for the initial levels of these characteristics and:

$$u_{it} = \rho_{ui}u_{it-1} + \epsilon_{it}, \quad |\rho_{ui}| < 1, \quad (39)$$

$$v_{it} = \rho_{vi}v_{it-1} + \epsilon_{it}, \quad |\rho_{vi}| < 1, \quad (40)$$

where:

$$\epsilon_{it} = \begin{cases} e_{i0} & \text{if } t = 0, \\ 0 & \text{otherwise.} \end{cases} \quad (41)$$

for $i = \{n, \gamma, R^*\}$. The three shocks can be related to the channels through which demography is linked with interest rates. The population growth rate shock affects labor supply. The surviving probability shock impacts expected retirement duration and, obviously, the world's interest rate shocks determine global interest rates.

4 Calibration

One period in the model corresponds to one year. Agents are assumed to enter the labor market at the age of 20 (we comment on this assumption in Section 6.5).

Following Carvalho *et al.* (2016), we calibrate the parameters of the world economy model to match the average characteristics of the developed countries. Therefore, most of the macroeconomic parameters are taken from Carvalho *et al.* (2016). The only difference is related to demographic shocks because of the variable retirement age and some minor modifications to the calibration procedure: Carvalho *et al.* (2016) calibrate demographic shocks to match the projected demographic characteristics at the end of the projection period. We minimize the sum of squared differences between the projected and the model-based characteristics during the whole period.

We assume that the shocks hit analyzed economies in 2000. To calibrate the demographic shocks we use the observed and projected characteristics for the period 2000–2100 presented in figure 1. The parameters governing the initial levels of the demographic variables are set to match the values observed in 2000. The parameters related to the steady state values of the macroeconomic characteristics are equal to the averages of the observed values in longer periods around the year 2000. The values of the parameters are presented in table 1.

4.1 Demographic characteristics

The initial retirement probability $1 - \omega_0$ is set to match the average effective retirement age according to OECD data for the year 2000 (<http://www.oecd.org/els/emp/average-effective-age-of-retirement.htm>; for the developed countries, we use the population-weighted average). The average effective retirement age was equal to 62.2 years in the developed countries and 60.4 years in Poland. This implies the retirement probability of 0.0237 and 0.0248, respectively. The initial population

Table 1: Baseline calibration of the model

Symbol	Description	Value	
		World	Poland
Demographic characteristics			
$1 - \omega_0$	initial retirement probability	0.0237	0.0248
n_0	initial population growth rate	0.0033	0.0005
γ_0	initial survival probability	0.9191	0.9139
ρ_{un}	autocorrelation of the u_{nt} process	0.9898	0.9904
ρ_{vn}	autocorrelation of the v_{nt} process	0.9908	0.9912
$\rho_{u\gamma}$	autocorrelation of the $u_{\gamma t}$ process	0.9902	0.9910
$\rho_{v\gamma}$	autocorrelation of the $v_{\gamma t}$ process	0.9915	0.9918
e_{n0}	realization of the ϵ_{n0} shock	0.0717	0.2575
$e_{\gamma 0}$	realization of the $\epsilon_{\gamma 0}$ shock	-1.0730	-1.4590
ι	sensitivity of ω_t to γ_t	0.0715	0.0796
Preferences and technology			
β	discount factor	1.0096	1.0009
$1 - \sigma$	inverse of elasticity of intertemporal substitution	0.5	0.5
α	capital share	0.333	0.350
δ	capital depreciation rate	0.1	0.1
x	technology growth rate	0.01	0.02
Government characteristics			
g	share of government consumption in GDP	0.2	0.183
b	government debt to GDP	0.6	0.461
E	aggregate pension transfers to GDP	0.0325	0.0475
World interest rate and risk premium			
ϕ	degree of economy openness	—	0.036
ρ_{uR^*}	autocorrelation of the u_{R^*t} process	—	0.9916
ρ_{vR^*}	autocorrelation of the v_{R^*t} process	—	0.9909
e_{R^*0}	realization of the ϵ_{R^*0} shock	—	-0.9177

growth rates n_0 are equal to the average population growth in the period 1995–2000. According to the United Nations demographic data, the rates are 0.33% for the developed world and 0.05% for Poland. The initial survival probabilities are calibrated to match the observed retirement-age dependency ratio in 2000 defined as population over the average effective retirement age to population between 20 and the average effective retirement age, given the population growth rate and the retirement probability (see formula (3)). In 2000, the dependency ratio was 28.2% in the developed countries and 28.6% in Poland. Thus, the initial values of the survival probabilities are equal to 0.9191 and 0.9139, respectively.

The characteristics of the exogenous shocks to the population growth rates (ρ_{un} , ρ_{vn} , e_{n0}) are set to match the observed and projected dynamics of the population. We minimize the distance between the theoretical and empirical time paths of these characteristics using the standard quadratic loss function.

The procedure of setting the characteristics of the shock to the survival probabilities $(\rho_{u\gamma}, \rho_{v\gamma}, e_{\gamma 0})$ together with the sensitivity parameter ι consists of the following steps:

1. Assume that the retirement age rises from the average effective retirement age in 2000 to 68 years in both economies in 2100;
2. Given the evolution of the retirement age from the previous step, calculate the retirement-age dependency ratio for the demographic data in the period 2000–2100;
3. Set the shock characteristics $(\rho_{u\gamma}, \rho_{v\gamma}, e_{\gamma 0})$ to minimize the square distance between the dependency ratio in the data (step 2) and the dependency ratio in the model described by equation (3);
4. Taking $\rho_{u\gamma}, \rho_{v\gamma}, e_{\gamma 0}$ from the previous step as given, set ι to minimize the difference between the simulated value of ω_{2100} in the model in 2100 (see formula (4)) and the value that implies the average retirement age of 68 years in 2100;
5. Repeat steps 2-4 until convergence is achieved.

The resulting theoretical retirement age in both economies is presented in figure 2, while projected and simulated demographic characteristics are included in both panels of figure 3.

The model's characteristics match their observed and projected counterparts quite well. However, some non-negligible discrepancies can be noticed, especially in the case of Poland where the demographic trends suddenly change around 2070. We address the impact of the imperfect fit on the simulation results in section 6.4.

4.2 Preferences and technology

As already mentioned, we take the values of the macroeconomic parameters for the world economy from Carvalho *et al.* (2016). Therefore, we use the standard values of the elasticity of intertemporal substitution $\sigma = 0.5$ and the technology growth rate $x = 0.01$. The capital share is set to $\alpha = 0.333$ and the annual capital depreciation rate to $\delta = 0.1$. The discount rate β is calibrated to match the steady state value of the interest rate of 3% (Carvalho *et al.* (2016) used the value of 4% in their simulations) which gives $\beta = 1.0096$.

As far as Poland is concerned, we use the same values for the elasticity of intertemporal substitution and the depreciation rate. The annual technological growth rate is set to 2%, twice the value for the developed countries. To pin down the capital share α in Poland we use data on the labor share of output provided by OECD database. The average capital share for the period 1992–2005 is close to 0.35. Finally, we assume that the steady state level of the interest rate in Poland is 2 p.p. higher than in the developed countries and set $\beta = 1.0009$. In the literature, the discount factor

Figure 2: Theoretical retirement age used in calibration of the demographic shocks

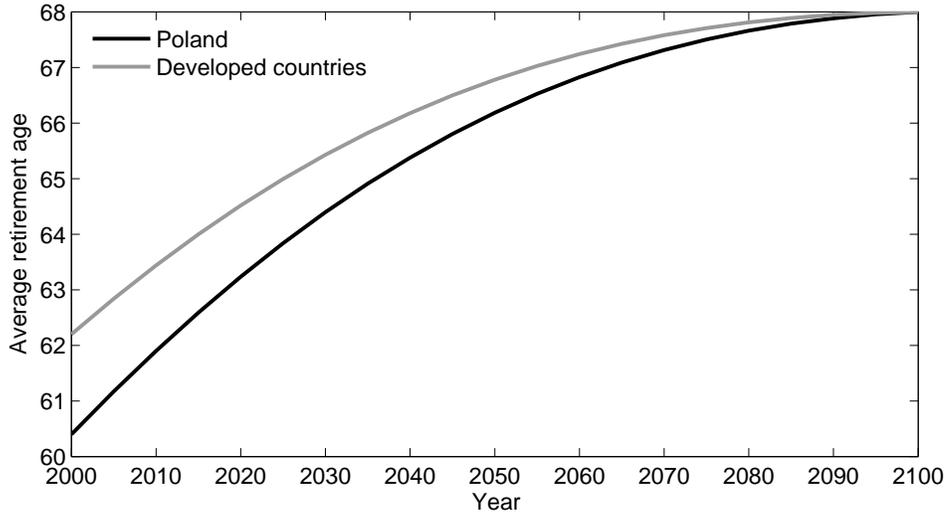
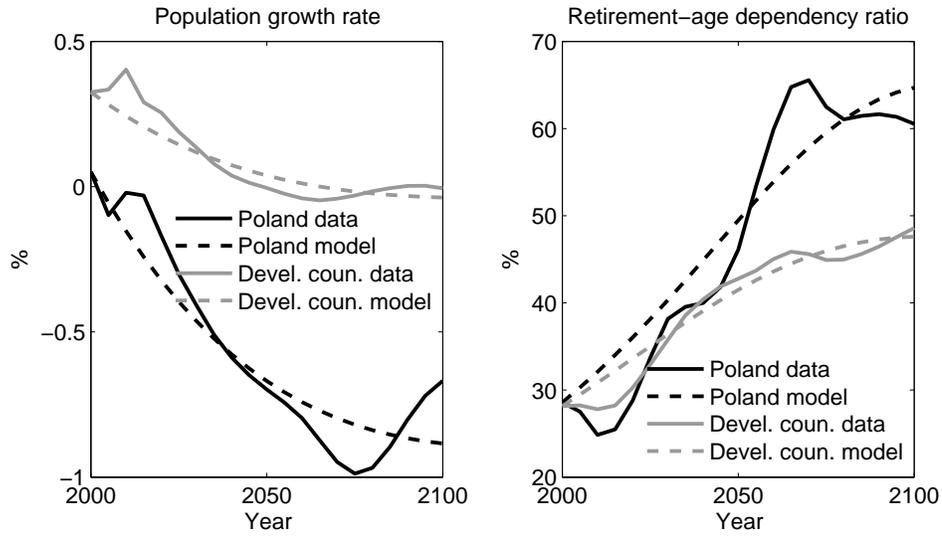


Figure 3: Projected and simulated demographic characteristics



β usually does not exceed unity. However, detailed sensitivity analysis for β lower than one conducted by Heer, Irmen (2014) reveal that apart from unrealistically high interest rate the results are practically unaffected.

4.3 Government characteristics

All the three parameters related to the government in the model for developed countries are also taken from Carvalho *et al.* (2016). They are equal to: $g = 0.2$, $b = 0.6$, and $E = 0.0325$. The first two parameters are directly pinned down by the Eurostat data. Therefore for Poland in the period 1995–2016 we have $g = 0.183$ and $b = 0.461$. Finally, we follow Carvalho *et al.* (2016) in calibrating the aggregate pension transfers E to GDP and get the values of 0.0475.

4.4 World interest rate shock and risk premium

To calibrate the parameters related to the foreign sector in the model for Poland, we first simulate the model for the developed countries. We use the projected dynamics of the world interest rate to estimate the parameters governing the world interest rate shock (ρ_{uR^*} , ρ_{vR^*} , e_{R^*0}). The standard quadratic loss function between the two trajectories is utilized in the estimation procedure. Finally, the parameter $\phi = 0.036$ governing openness of the economy is set to match the average fraction of the net foreign assets to GDP in Poland (F/Y) equal to 55%.

5 Main results

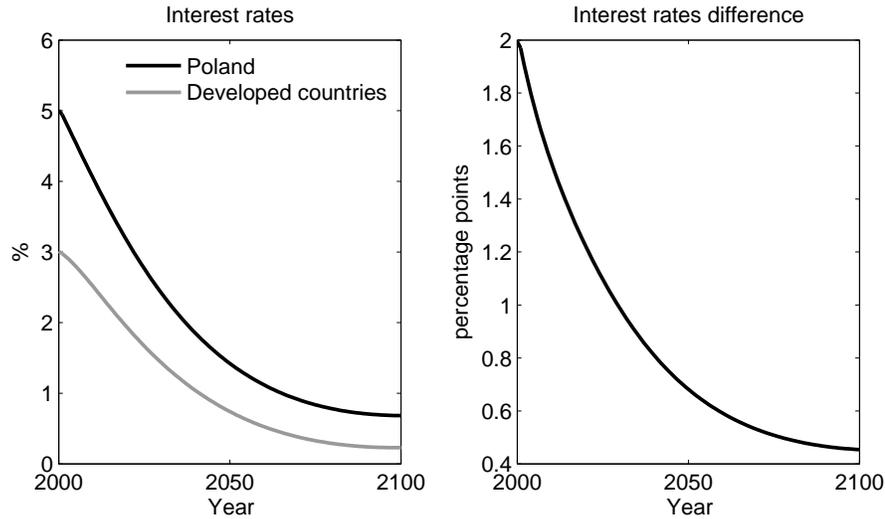
In this section, we present the results of simulations of the model. First, we focus on the projected dynamics of the interest rates and other macroeconomic aggregates. Then, we examine sensitivity of the interest rate evolution to changes in the effective retirement age.

5.1 Dynamics of the interest rates and the macroeconomic aggregates

Figure 4 depicts projected dynamics of the interest rates in Poland and developed countries. In line with the results presented in the literature, we show a sharp decline in the interest rates in both economies. At the end of the century, they will be barely positive. In Poland the drop is expected to be larger than in the developed countries. As a result, the interest rate gap will also decline and reach 0.5 p.p. at the end of the simulation period. Presented numbers are very close to those obtained by Bielecki *et al.* (2017) although they assume quite different initial values of the world and the national interest rates at the beginning of the simulation period equal to 1.25% and 2.25%, respectively.

Figure 5 shows the results for the other macroeconomic aggregates. It should be noted that these projections do not account for the technological progress which is subject to high uncertainty and its projection remains outside the scope of this study. However, as shown in the next section, the rate of technological progress has a small

Figure 4: Simulated dynamics of the interest rates. The interest rate difference is defined as $R_{Poland} - R_{World}$



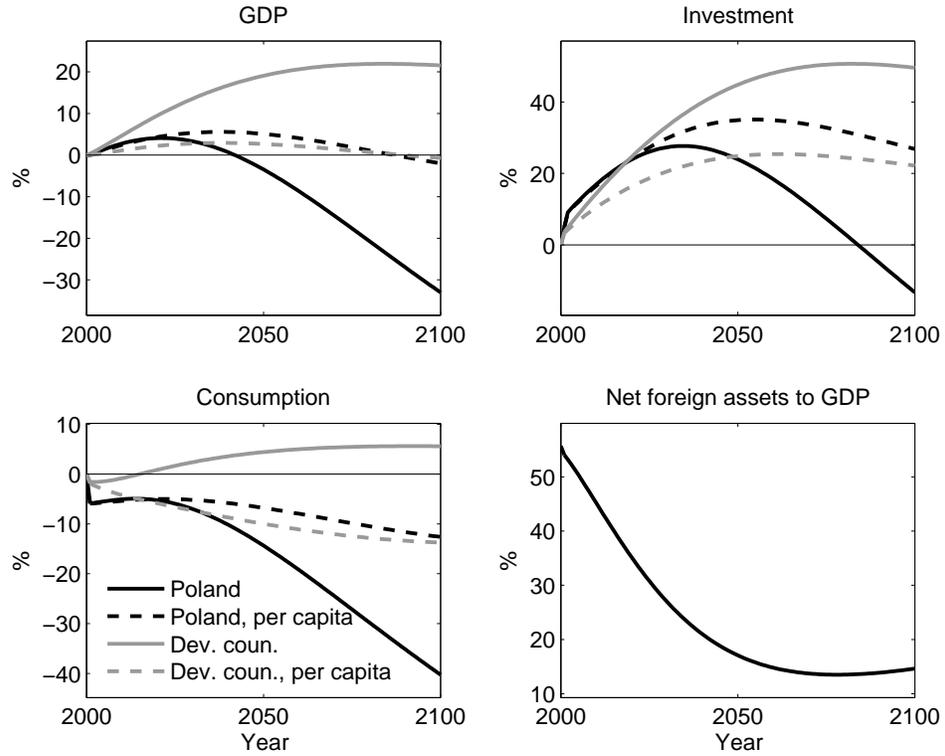
impact on the interest rates.

Because of the demographic changes, the GDP level in the developed countries is expected to rise by 20% in the simulation period. At the same time, it will drop by 30% in Poland. The difference simply reflects the projected difference in the total population as the per capita quantities will behave very similarly in both economies. The decrease in the interest rates will be associated with the boom in investment. In the developed countries, its level is expected to rise by 50%. In Poland, the rise will reach 30% about 2030 and then, due to decreasing population, investment will begin to decline. The per capita quantities should increase by 20-30% in both economies. The higher capital accumulation will result in lower consumption. The drop is projected to reach about 10% in per capita terms. The drop will be severely deeper in Poland and compensated in the developed countries by changes in total population. Finally, the declining interest rate gap will improve the net foreign asset to GDP ratio in Poland which is projected to drop up to 15%.

5.2 Role of the retirement age

Projected dynamics of the interest rates is sensitive to changes in the effective retirement age. The evolution of the interest rates under different values of the retirement age in 2100 is summarized in figure 6. The left plot shows the interest rate levels in 2100, whereas the right one depicts the average rates for the whole simulation period. The figure documents a strong positive relationship between interest rates and

Figure 5: Projected dynamics of the main macro aggregates. All the variables except net foreign assets to GDP are expressed as percentage deviations from the initial values. The projections do not account for the technological progress x



the retirement age. As far as the interest rate level in 2100 is concerned, a one-year increase in the effective retirement age is associated with a rise in the interest rate level of approximately 0.11 p.p. in Poland and 0.13 p.p. in the developed countries. For mean interest rates, these values are equal to 0.14 p.p. and 0.13 p.p., respectively. Figure 7 shows interest rate gaps for different values of the effective retirement age. Depending on the retirement age policy, the gap in 2100 can range from -0.5 to 1.5 p.p. The mean gap for the whole simulation period is expected to vary from -0.4 to 2 p.p. Our results are somehow different than those of Sudo, Takizuka (2018) who simulate evolution of interest rates and macro aggregates for three different paths of the retirement age and find a relatively weak impact of these institutional changes as compared with that of total factor productivity. Nevertheless, their analysis is based on a closed-economy version of the model and calibrated for Japan which has already experienced its most dramatic demographic developments.

Figure 6: Simulated interest rates for different values of the average retirement age in 2100

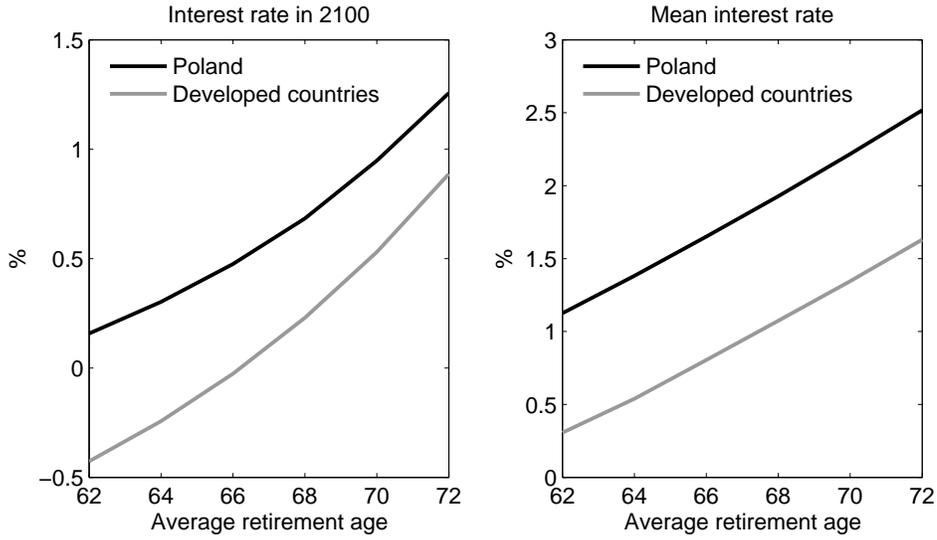
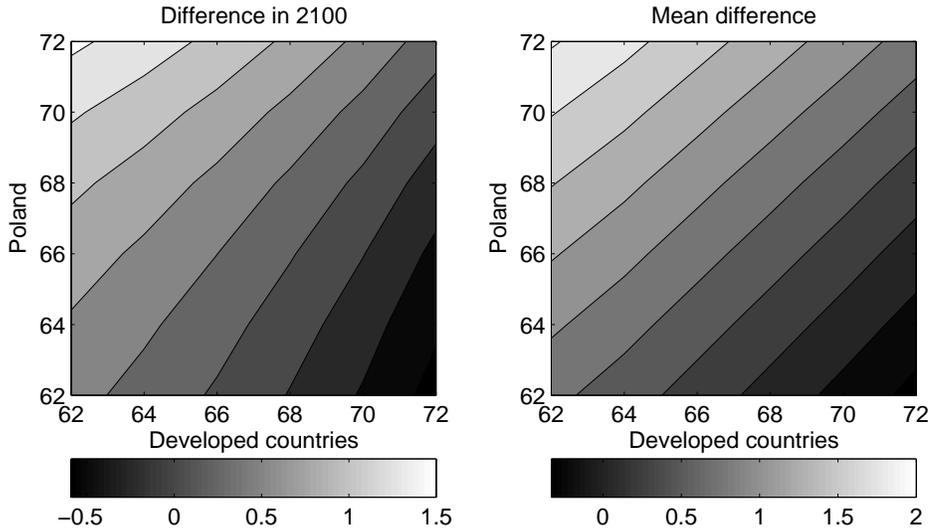


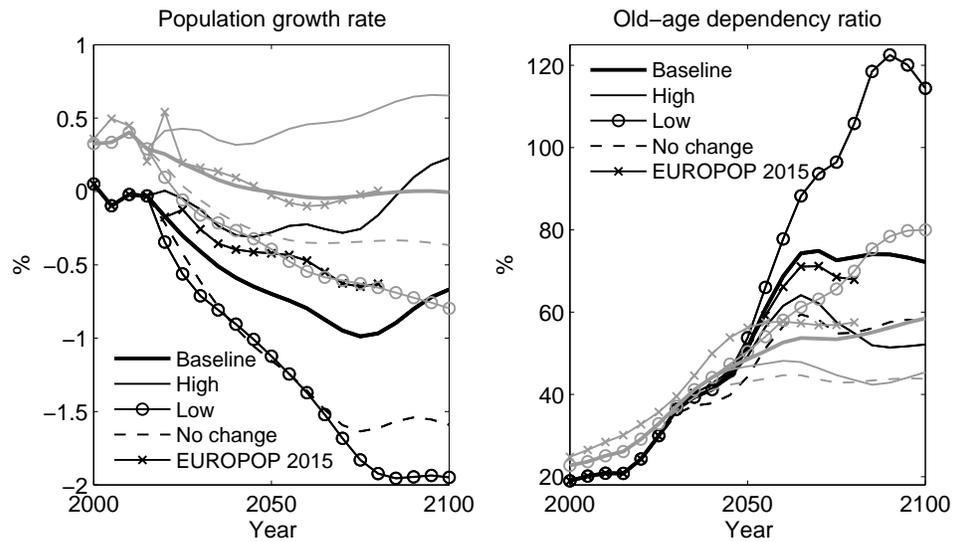
Figure 7: Projected interest rate differentials for different values of the average retirement age in 2100. The plots illustrate the projected interest rate difference: $R_{Poland} - R_{World}$ in percentage points



6 Additional results

This section presents some additional results enriching conducted analysis. First, we consider alternative population projections provided by the United Nations and the Eurostat. Then, we discuss the role of particular shocks and technical progress for interest rate dynamics. Finally, we analyze the impact of an additional shock introduced to match better simulated demographic characteristics and we estimate the effects of changes in the labor market entrance age.

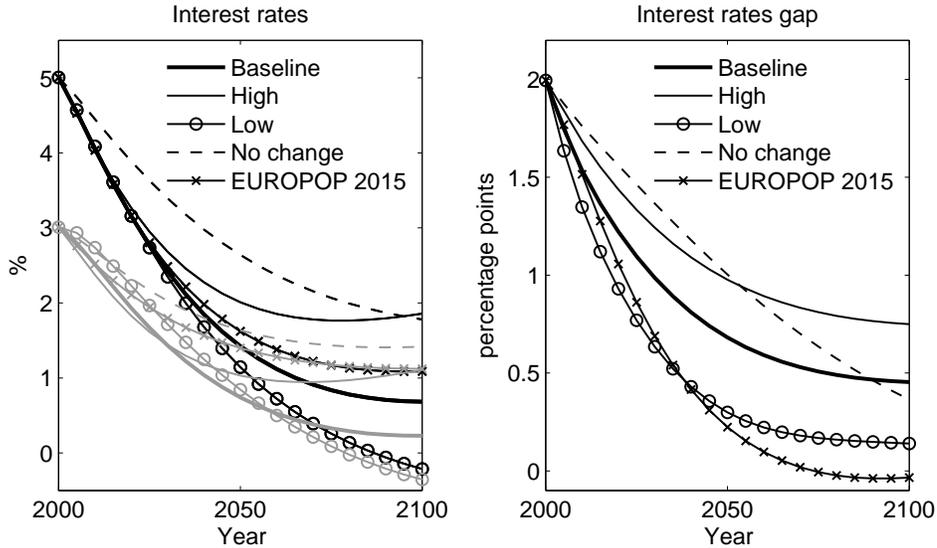
Figure 8: Alternative variants of the projected demographic characteristics. The projections for Poland are marked with black lines and for the developed countries — with grey



6.1 Alternative population projections

Four alternative population projections are studied here. Three of them are the alternative variants to the medium scenario of the UN latest projection. These three variants — called high, low, and no change — differ primarily in terms of projected fertility. In the first two cases the normal projected mortality is assumed, whereas in the last variant constant mortality is used. The detailed description of the assumptions can be found in the publication of the United Nations (2017). The fourth projection is the baseline scenario of the EUROPOP2015 projection prepared

Figure 9: Simulated interest rate dynamics for the alternative population projections. Black lines refer to Poland, while the grey ones to the developed countries



by the Eurostat. This projection is prepared for the whole European Union and single countries. In the case of the developed countries, we aggregate available projections for 18 members of the euro area.

Projected population dynamics as well as the old-age dependency ratio are depicted in figure 8. As expected, the highest population growth rates are projected in the high variant, although even in this case the population growth in Poland is barely positive. The most severe depopulation is expected in the low and no change variants with the decline rates in Poland reaching even 2% per year. The EUROPOP projection for the developed countries coincides with the UN medium scenario, whereas for Poland it is slightly more favorable.

Similar conclusions apply to the old-age dependency ratio. The highest values are expected in the low fertility variants, while the lowest ones are forecasted in the high and no change scenarios. The EUROPOP projections give slightly higher ratios compared to the medium scenario in Poland and slightly lower in the developed countries.

Simulated dynamics of the interest rates and their gaps under alternative demographic projections are depicted in figure 9. Interest rates are expected to drop significantly regardless of the considered variant, although the magnitudes and trajectories of the declines differ considerably. In the low fertility variant the interest rates in Poland are expected to drop below 0% in 2100, whereas in the case of the high and no change scenarios it will decrease to 2% only. Subsequent interest rate gaps are projected to

vary from 0 p.p. under the EUROPOP variant to 0.75 p.p. under the high fertility scenario.

Figure 10 depicts the projected consequences for major macro aggregates. While different variants deliver quite dispersed projections regarding their levels that reflect expected changes in total population, per capita forecasts are rather homogeneous. Consumption is the only exception here. All but the no change variants predict a small but steady decline in per capita consumption. However, under the no change scenario, this variable is also expected to stay close to the initial level.

6.2 Impact of particular shocks

In this subsection, we examine the role played by particular shocks for the baseline results. We disentangle these effects by simulating the model with one shock being eliminated. The results presented in figure 11 indicate clearly that the expected sharp decline in the interest rate in Poland is primarily driven by the surviving probability shock. The interest rate will drop by less than 1 p.p., if the life expectancy does not rise. In other words, the decline in the interest rate will be caused mainly by the increase in savings related to the longer expected retirement duration. The impact of the remaining shocks is much smaller. The drops in the interest rate due to the decreasing global interest rates and the decreasing labor supply are less than 0.3 p.p. and 1 p.p., respectively.

6.3 Convergence of the technological growth rates

Throughout the paper we assume that the technological progress in Poland is fixed at 2% and is 1 p.p. higher than in the developed countries during the whole simulation period. Of course, it would be more realistic to assume that technological progress in Poland will be slowing down and some form of convergence will be observed. However, in this subsection we show that changes in the rate of technological progress have a minor impact on the interest rate dynamics.

Figure 12 compares projected dynamics of the interest rates in Poland under different, constant rates of technological progress equal to 2% (the baseline calibration) and 1%, respectively. Generally, the difference between the trajectories is small and never exceeds 0.5 p.p. More precisely, lowering the rate of technological progress decreases the steady state level of the interest rate which generates the gap between the trajectories at the beginning of the simulation period. On the other hand, the interest rate is less sensitive to the demographic shocks under the lower rate of technological progress and the decline in the interest rate is lower compared to the baseline case. As a result, the initial gap disappears and the two trajectories coincide. Therefore, if we assume that the rate of technological progress slowly converges to 1% the projected interest rate trajectory should be even closer to the baseline case.

Figure 10: Simulated dynamics of the main macro aggregates for the alternative population projections. All the variables but the net foreign assets to GDP are expressed in percentage deviations from the initial values. The projections do not account for the technological progress x . Black lines refer to Poland, while the grey ones to the developed countries

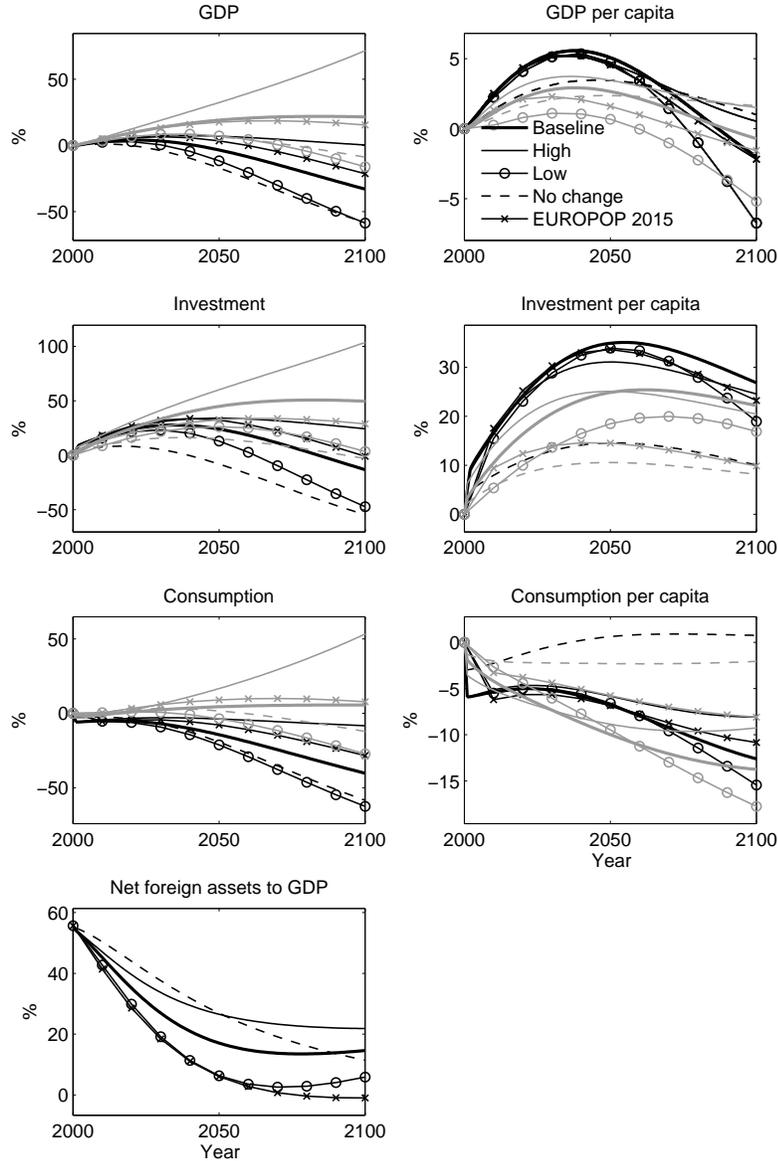


Figure 11: Simulated interest rate in Poland without the particular shocks

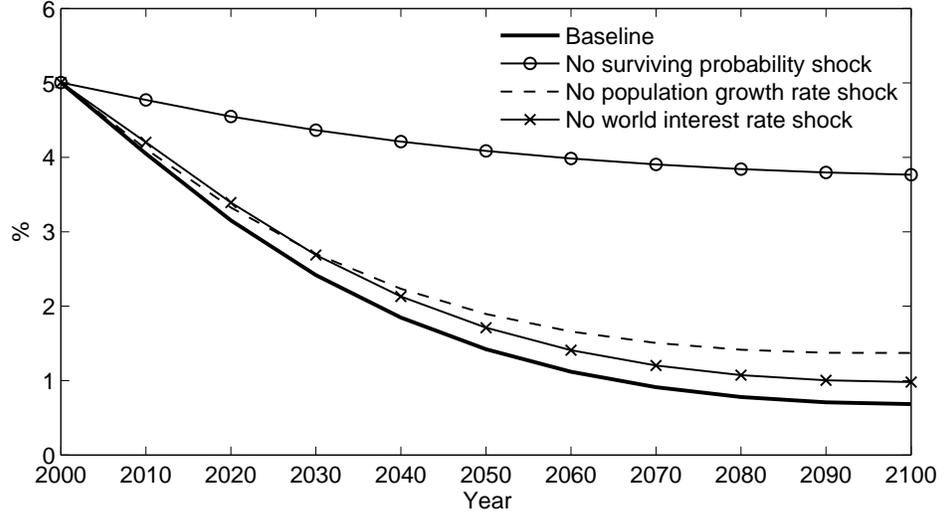


Figure 12: Simulated interest rate dynamics in Poland for different values of the technological growth rate parameter x

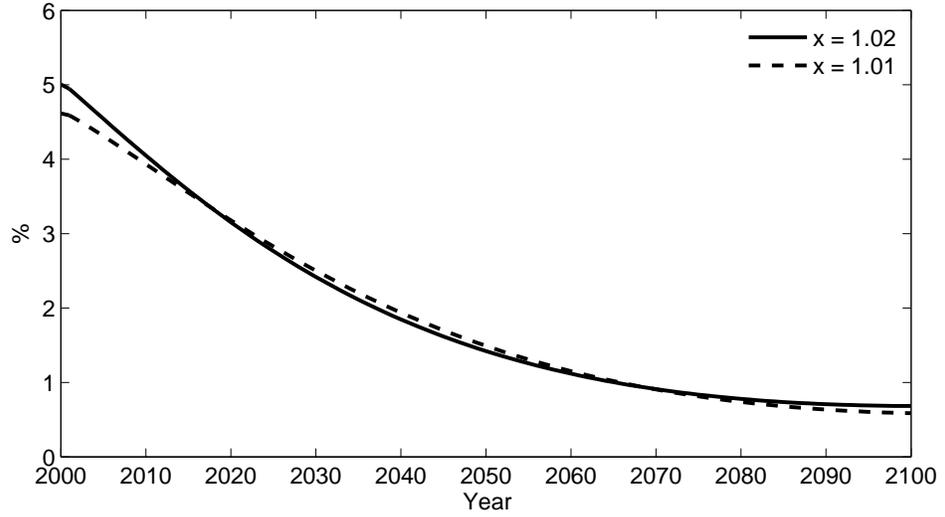


Figure 13: Projected and simulated demographic characteristics in case of additional innovations in the demographic shock processes

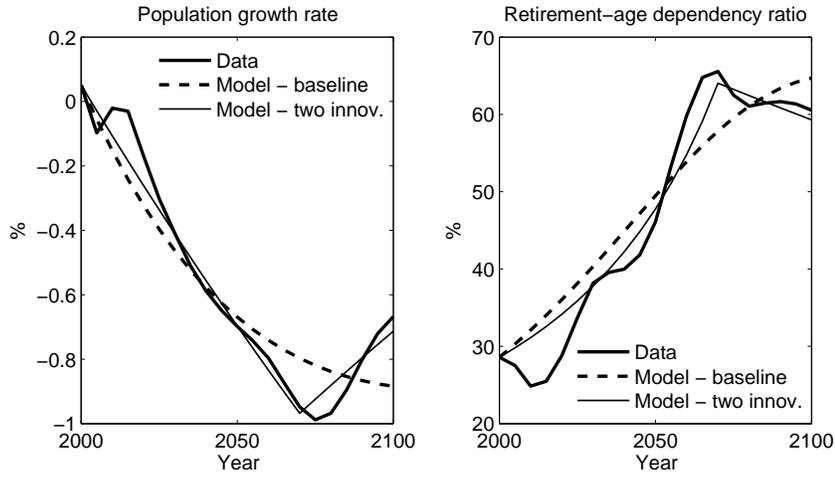
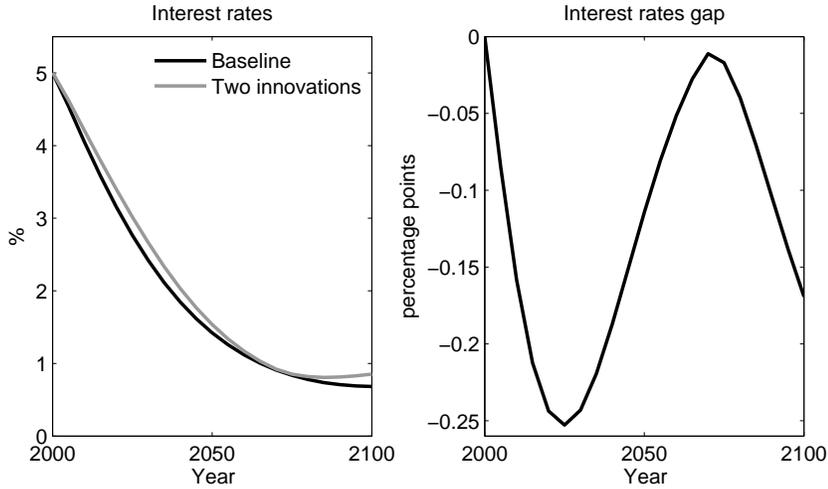


Figure 14: Interest rate dynamics with an additional demographic shock. The interest rate gap is defined as $R_{baseline} - R_{two\ innov.}$

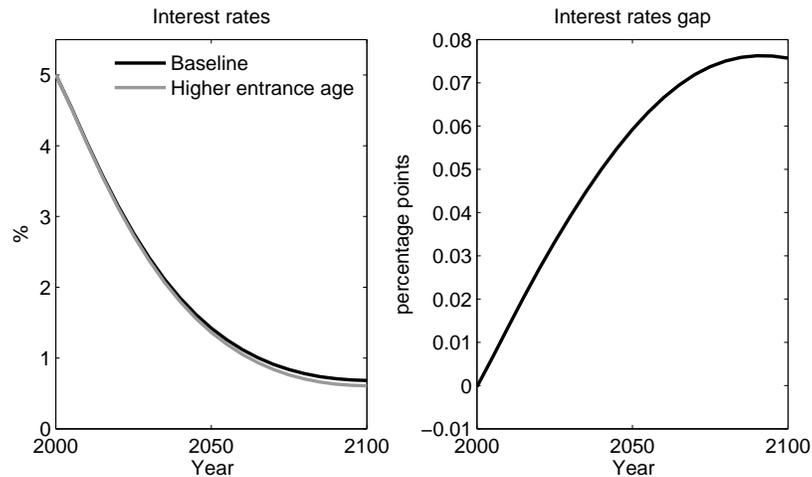


6.4 Improving the model's fit

As already mentioned, the theoretical dynamics of demographic characteristics does not match exactly their official forecasts. The model's fit in this regard can be easily improved by considering additional non-zero innovations e_{it} in the exogenous processes (39) for n_t and γ_t for Poland. These innovations are aimed to capture the sudden change in the demographic projections around the year 2070. We recalibrate the shock processes for Poland using the procedure described in subsection 4.1 with two non-zero innovations: one in the year 2000 and the second one in 2070.

A noticeable improvement in the model's fit is documented in figure 13. The model is simulated assuming that the value of the second shock is revealed to the households at the beginning of the simulation period (perfect-foresight simulations). The effect of the change for the projected interest rate dynamics is shown in figure 14. With the additional shock the drop in the interest rate is somewhat smaller. As compared to the baseline case, the difference is on average about 0.1 p.p. and never exceeds 0.25 p.p.

Figure 15: Simulated interest rate dynamics in Poland for the higher labor market entrance age. The interest rate gap is defined as $R_{baseline} - R_{higher\ ea}$



6.5 Role of the labor market entrance age

It should be noted that the results can be also sensitive to changes in the age at which households enter the labor market, although it is difficult to predict how exactly this age will evolve in the future. A potential increase in the labor market entrance age reduces the length of a labor market career, but leaves the retirement duration

unaffected. To examine the size of this effect, we change the retirement probability parameter ω_0 in Poland from the baseline value of 0.0248 to 0.0278 which corresponds to an increase in the labor market entrance age by four years. Simultaneously, the discount factor is adjusted, so that the initial value of the interest rate in Poland remains equal to 5%. As documented in figure 15, the effect of changes in the labor market entrance age is small and does not exceed 0.1 p.p. despite considering an immediate change. In reality, the entrance age is likely to rise gradually, so the difference would be even smaller.

7 Concluding remarks

Due to adverse demographics many developed countries are likely to experience significant declines in interest rates and profound changes in their macro aggregates. Economic consequences of an older population can be attenuated if confronted with individual responses (such as greater female labor-force participation), policy reforms (e.g. increase in the legal retirement age) (Bloom *et al.*, 2010), and — in case of open economies — immigration.

Presented estimates are subject to change when new population projections become available. Especially migration flows that can modify fertility, population growth and structure, are associated with high uncertainty. Immigration is not likely to induce substantial changes in fertility patterns as many immigrants converge to the behavior of natives, so that higher fertility of immigrants contributes only to a minor change in total fertility rate and differences in fertility decline over time (Frejka *et al.*, 2008). Nonetheless, because of its mercurial character and large variability migration can significantly alter population size and structure in analyzed economies.

Although we allow for migration (to the extent that this phenomenon is included in population projections) and capital flows, we abstract from international trade. Another limitation of our analysis refers to the assumptions concerning the statutory retirement age that has been recently reduced and the shape of the pension system which since 1999 has not been a standard PAYG system. Furthermore, we do not take into account numerous developing countries with a significantly different course of demographic processes which may impinge both on migration and capital flows on the global scale.

Policy recommendations for ageing economies could focus on the problem of a shrinking labor force and include implementation of measures aimed at increasing fertility and labor force participation rate, postponing retirement by working individuals and receiving a higher number of immigrants and homecoming migrants. However, the message of our paper is somewhat different. Our simulations do not indicate that lowering of interest rates in Poland due to ageing will have a detrimental effect on its economy. To the contrary, during forthcoming decades Poland is likely to experience much faster growth in investment per capita and in GDP per capita than other developed countries. We also demonstrate that faster ageing in Poland

naturally contributes to the convergence of interest rates between Poland and the rest of the world. As a consequence, the argument that it is the euro adoption that can contribute to the decrease in interest rates and greater investment in Poland gradually loses its strength.

Against this background, probably the most important issue for policymakers is how projected demographic processes will affect individual propensity to save. The question to what extent the government can rely on rationality and farsightedness of individuals on one hand and on already implemented measures like Employee Capital Plans or phasing out Sunday trading on the other hand, remains open for public debate.

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