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Youth unemployment and welfare gains from eliminating business cycles — The case of Poland

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ABSTRACT

In many countries, business cycle fluctuations of unemployment rate among youth are much higher than that of other cohorts. We investigate how this life-cycle heterogeneity of unemployment risk affects welfare gains from eliminating business cycles in Poland. We use an overlapping generations version of the heterogeneous agent model with aggregate risk and borrowing constraints. We find that accounting for life-cycle heterogeneity of unemployment risk can increase the gains even by 60%. The results also show that consumption of young cohorts drops due to business cycles at least a few times more than the average for the whole population.

1. Introduction

Many empirical studies show that recessions are particularly harmful for young people because the unemployment risk for this group rises much more than for other cohorts. In this paper, we quantify these costs in terms of welfare. More precisely, we study the welfare gains from eliminating business cycles, as well as their distribution across cohorts, while taking into account the life-cycle heterogeneity of unemployment risk.

The problem of the high relative youth unemployment rate as well as its excess sensitivity to business cycles is well documented in the literature (see for example Bruno et al., 2014, Hoynes et al., 2012, Jimeno and Rodriguez-Palenzuela, 2002, Kawaguchi and Murao, 2012). It is particularly severe in Central and Southern Europe. For example in Poland, during the period 1997–2013, the unemployment rate for the 20–24 age group soared, on average, from 24v in booms to 33% in downturns. At the same time, the rates for the 25–60 group were 9 and 13%, respectively. Similar jumps in youth unemployment have been observed in other countries, such as Spain, Bulgaria, Slovakia and Lithuania. The recent financial crisis has been particularly harmful for young people, not only because of the rapid increase in the unemployment rate, which for the whole OECD area rose by 6 percentage points (Bell and Blachflower, 2011; ILO, 2012; Scarpetta et al., 2010), but also because of the high persistence of unemployment. For example, in Greece and Spain, more than 40% of young are still unemployed.

While the welfare costs associated with the high level of youth unemployment have been investigated recently by Chéron et al. (2013) or Michelacci and Ruffo (2015), among others, there are no papers, at least to our knowledge, examining the welfare costs related to the high business cycle volatility of youth unemployment.

The methodology for calculating welfare gains from eliminating business cycles has been developing since the seminal contribution of Lucas (1987). His finding that the gains for an average consumer represent less than 0.01% of lifetime consumption has been challenged from various directions. One important strand of the critique argues that the idiosyncratic consumption risk faced by individuals is much higher than the aggregate data used by Lucas suggest (Atkeson and Phelan, 1994; Beaudry and Pages, 2001; Gomes et al., 2001; Imrohoroğlu, 1989; Krusell et al., 2009; Krusell and Smith, 1999; Mukoyama and Şahin, 2006; Storesletten et al., 2001). Some of the cited papers consider not only the gain for an average agent but also study a distribution of the gains across agents. They identify some

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groups of people, especially the poor (Krusell et al., 2009; Krusell and Smith, 1999), the low-skilled (Mukoyama and Şahin, 2006) and the young (Storesletten et al., 2001), for whom the gains are much higher than the average for all agents.

The gain for the poor stems from their inability to insure themselves against unemployment risk, which rises periodically due to business cycles. The high gain from eliminating business cycles for low-skilled workers results mainly from the much higher unemployment risk they face during recessions compared to skilled workers. However, only Storesletten et al. (2001) analyze the life-cycle distribution of the gains. Nonetheless, they do not account for the life-cycle heterogeneity of idiosyncratic risk. In fact, in their setup, the low wealth level coupled with the precautionary motive related to high uncertainty about lifetime earnings is the logic behind the severity of business cycles for young agents.

In this paper, we build on the work of Rios-Rull (1994) and develop an overlapping generations version of the heterogeneous agent model used by Krusell and Smith (1999) and in particular by Mukoyama and Şahin (2006) to account for the life-cycle heterogeneity in unemployment risk. In our setup, agents differ in terms of wealth, skills, labor market status and age. They are subject to idiosyncratic labor market risk. The transition probabilities between employment and unemployment depend not only on an agent's skill level and the aggregate shock but also on age. The model is calibrated using data for the Polish economy. In particular, the transition probabilities are set to match the age profiles of average unemployment rates and durations for workers with different skill levels in booms and recessions.

There are two reasons for using data from Poland. First, as noted earlier, the life-cycle heterogeneity of unemployment risk in Poland is high. However, more importantly, the mean duration of unemployment exceeds one year. This allows us to set up the OLG model at an annual frequency, which considerably facilitates the computations. Besides that, there is one additional feature that makes our baseline calibration of the model unusual, namely, flat unemployment benefits, which in Poland are generally the same for all workers regardless of their previous earnings. However, we also consider a calibration with proportional unemployment benefits. Thus, apart from the unemployment duration and the benefits, our calibration does not differ much from what is used in the literature. Therefore, we think that the insights gained from our study would also apply to other economies with a high volatility of youth unemployment risk.

We consider the welfare gains from two perspectives: a one-period, or momentary, utility of a single cohort and the lifetime welfare of a group of newborn agents. We use the latter perspective as a finite lifetime analogue to the standard measure introduced by Lucas (1987). In our paper, the lifetime gain is defined as a constant percentage increase in consumption of a group of newborn agents in the economy with business cycles needed to equalize the average expected lifetime utility for the agents in the economy with and without business cycles. The momentary gain is calculated in a similar manner, but now we equate the average momentary utilities for certain cohorts. Calculating the welfare gains, we explicitly take into account the transition from an economy with the aggregate risk to the world without it.

To assess the welfare gains from eliminating business cycles, we have to consider a hypothetical economy without business cycles. In particular, one should decide to what extent the idiosyncratic risk is affected by removing the aggregate risk. Due to computational difficulties, we do not apply the integration principle advocated by Krusell and Smith (1999) and Krusell et al. (2009). Instead, we follow Reiter (2012) and consider two possibilities. We assume that the transition probabilities in the economy without business cycles either are set to match the means of unemployment levels and durations for booms and recessions, as in İmrohoroğlu (1989), or simply equal averages of the respective transition probabilities for the two states of the economy. The latter approach is a natural consequence of the non-linearity of the unemployment rate, as recently shown by studies employing search and matching labor market models (Hairault et al., 2010; Iliopulos et al., 2014; Jung and Kuester, 2011; Petrosky-Nadeau and Zhang, 2013). It should also be mentioned that consequences of stabilizing business cycle fluctuations on labor market are also studied by the optimal unemployment insurance literature (Landais et al., 2010; Mitman and Rabinovich, 2015).

Our findings can be summarized as follows. Under a conservative parametrization, the gains from eliminating business cycles for young cohorts are at least two to four times higher than the average for the whole population. In other words, the decline in consumption caused by business cycles is at least a few times higher for young agents. Additionally, we identify a few other reasonable parametrizations where the differences are even more spectacular. We also show that the life-cycle heterogeneity of unemployment risk, disregarded by previous studies, significantly increases the lifetime gains for all type of agents. High volatility of unemployment risk coupled with reduced abilities to self-insure against unemployment risk plays a key role in explaining the fact that majority of the costs associated with business cycles is borne by young agents. We document that eliminating the business-cycle variation in the labor market risk for cohorts 20–29 exclusively can reduce the lifetime gains by as much as 90%. Due to the flat unemployment benefits in Poland, the gains are generally highest for high-skilled workers. This, however, is no longer the case for proportional unemployment benefits.

The rest of the paper is organized as follows. In Section 2, we introduce the model used for studying the gains from eliminating business cycles. Then, in Section 3, we present the calibration of the model's parameters focusing on matching the life-cycle heterogeneity of unemployment risk. Section 4 discusses the issues related to the measurement of the welfare gains within the framework described in Section 2. Results of various numerical simulations are presented in Section 5. Finally, in Section 6, we conclude the paper and briefly discuss two factors excluded from our considerations that are likely to affect the findings of the study.

2. Model

We use an overlapping generations version of the heterogeneous agent model of Mukoyama and Şahin (2006). However, we slightly depart from their setup in a few points. First, because we are not interested in matching wealth distributions, we use the same discount factor for all agents. Second, we allow for negative wealth holdings. We think that the no-debt requirement is an unrealistic assumption in life-cycle frameworks, which, as shown below, considerably increases the welfare gains for young cohorts. Finally, we assume that agents cannot change their skill levels.

2.1. General setup

The economy is populated by a continuum of finitely lived agents who differ in terms of age a, skill level s, employment status ε and wealth k. For simplicity, we omit the time subscripts and use primes to denote subsequent periods. Agents enter the labor market at the age of 20, work for 40 years, then retire and live, at most, up to 99 years. The life length is stochastic.

Young agents either work ($\varepsilon = e$) or are unemployed ($\varepsilon = u$). If employed, they supply $\xi_s \xi_a l$ effective units of labor and obtain the net income $(1-\tau)\xi_s\xi_a lW$, where τ is the tax rate, W stands for the aggregate wage, l is the constant for all agents' nominal labor supply and ξ_s and ξ_a denote the labor efficiency factors related to skill level and age. The unemployed agents receive unemployment benefits. In the baseline version of the model, we assume that the benefit is proportional to the mean wage in the economy and, therefore, is equal to $\theta_u(1-\tau)\bar{\xi}lW$, where θ_u is the unemployment replacement rate and

$$\bar{\xi} = \sum_s \sum_a \left[1 - 0.5\left(\bar{u}(s,a,Z_b) + \bar{u}(s,a,Z_g)\right)\right]\xi_s \xi_a \omega(s,a) \tag{1}$$

denotes the mean labor efficiency across agents, $\omega(s,a)$ is the fraction of agents with skill level s and age a, $\bar{u}(\cdot)$ represents the unemployment rate for a given cohort and Z is the aggregate stochastic shock. Retirees receive pensions that are proportional to the wage of an employed agent of age 59 $\theta_r(1-\tau)\xi(s)\xi_{59}lW$, where θ_r represents the pension replacement rate. As a result, an agent's work-related income d is given by:

$$d = (1-\tau)lW\left[\xi_s\xi_a \cdot 1(a<60,\varepsilon=e) + \theta_u\bar{\xi}\cdot 1(a<60,\varepsilon=u) + \theta_r\xi_s\xi_{59}\cdot 1(a\geq 60)\right], \tag{2}$$

where $1(\cdot)$ stands for the indicator function. Moreover, the agents receive interest R on their capital stock. We allow for negative wealth holdings, but debt cannot exceed some prespecified level $\underline{k}(s)$. In the baseline version of the model, we assume that newborn agents start with zero capital stock and wealth of agents who die is immediately spent on public consumption by the government so bequests do not enter agents' decision problems. For the robustness check, we will also consider a situation where wealth of agents who die is inherited by newborn consumers.

The production sector consists of one representative firm that hires capital and labor from agents and produces a single consumption good according to the standard Cobb–Douglas technology:

$$Y = ZK^{\alpha}L^{1-\alpha}, \tag{3}$$

where K and L are aggregate capital and aggregate effective labor, respectively:

$$K = \int k_j dj, \quad L = l\bar{\xi}, \tag{4}$$

and k_j represents the capital stock of the j-th agent. Because the firm operates on competitive market, it sets the aggregate wage and interest rate equal to the marginal products of labor and capital:

$$W = (1-\alpha)ZK^{\alpha}L^{-\alpha}, \quad R = \alpha ZK^{\alpha-1}L^{1-\alpha}. \tag{5}$$

Every period, an agent faces the standard consumption-saving problem, which can be recursively written as:

$$V(k,99,\varepsilon,s,K,Z) = U(k+d), \tag{6}$$

$$V(k,a,\varepsilon,s,K,Z) = \max_{c,k'}\{U(c) + \beta q_{a,a+1}\mathbb{E}\left[V(k',a+1,\varepsilon',s,K',Z')\,|\,\varepsilon,K,Z\right]\} \tag{7}$$

$$\text{s.t.} \quad k' = (1-\delta+R)k + d - c, \tag{8}$$

$$k' \geq \underline{k}(s), \quad K' = H(K,Z,Z'), \tag{9}$$

where $V(\cdot)$ is an agent's value function, β is a discount coefficient, $q_{a,a+1}$ denotes the one-year surviving probability for an agent of age a and $H(\cdot)$ represents a law of motion for aggregate capital.[1] As the instantaneous utility function, we use the standard CRRA utility:

$$U(c) = \frac{c^{1-\gamma}-1}{1-\gamma}, \tag{10}$$

where γ is the risk aversion coefficient.

There is also a government in the model that imposes taxes on the income from work to finance the unemployment benefits and pensions. The tax rates are set so the government budget is always balanced. The exact derivation of the tax rates is presented in the appendix.

2.2. Stochastic structure of the economy

There are three exogenous stochastic shocks in the model. The aggregate productivity shock Z is represented by a two state Markov chain with the transition matrix P_Z. The states $Z = \{Z_b, Z_g\}$ represent recession and expansion period, respectively. The individual employment shock ε is also modeled as a two state Markov chain with the transition matrix $P_\varepsilon(s,a,Z,Z')$. Here, the transition probabilities depend on the current and future state of the economy as well as an agent's skill level and age. Finally, a lifetime in the model is stochastic. For every cohort a, there is a fraction $1-q_{a,a+1}$ of agents who die.

2.3. Additional remarks

In the paper, we neglect the impact of the world economy and use the closed economy setup. While this assumption is far from the reality, we believe that it does not affect significantly the results of the study. This is because we focus on the labor market status of the individuals and its impact on income and consumption of agents which are primarily driven by domestic forces. Moreover, the employment status of agents and the aggregate state of the economy are determined exogenously and, as discussed in the next section, with the proper calibration, the role of the rest of the world can be accounted for, at least partially.

Under assumption that agents have access to international capital markets their budget constraint would be different. The assumption would also affect the domestic interest rate. However, the budget constraint channel is likely to be weak because of the high domestic bias in capital market observed in emerging economies (see for example Fiodra et al., 2007). The effect on interest rate would probably be more important. High synchronization of business cycles in Poland and the rest of the world raises the volatility of the domestic interest rate which, in turn, increases the gains from eliminating business cycles.

3. Calibration

One period in the model corresponds to one year. Although some authors also employ the yearly specification (see Storesletten et al., 2001), it is a rather rare choice regarding the cost of business cycles. However, we have at least two reasons for using this approach. First, it reduces the computational complexity of the problem. In the quarterly model, there would be 320 cohorts instead of 80 for the yearly specification. Second, it allows for a more realistic calibration of the labor market transition probabilities. This virtue is the consequence of how we construct the transition matrices. They are built

[1] In our notation, the value function V depends on aggregate capital K. Actually, this is a simplification stemming from the Krusell–Smith algorithm used for approximating a solution to the decision problem. More details are given in the technical appendix.

in such a way that for each cohort there are only two different values of the unemployment rate — one for the recession and one for the expansion. When the aggregate state of the economy switches, the unemployment rate immediately jumps to the new level. It is much more realistic to assume that such adjustment is completed within one year than one quarter, to put it simple. Or more precisely, if the transition probability from unemployment to employment is pinned down by the average duration of unemployment, then one has to assume extremely low probability of losing a job to appropriately decrease the unemployment rate within one period. In fact, in many cases, it is simply impossible with a quarterly calibration (this probability should be negative). Of course, the yearly specification also means that the unemployment duration in the model is a multiple of a year, which is far from what we observe in the data.

We divide the parameters into two groups. In the first subsection, we describe the calibration of the structural parameters. Then, we move to the parameters governing the stochastic structure of the model.

3.1. Structural parameters

The baseline calibration of the structural parameters is presented in Table 1. The capital share in the production function α is set at 0.41. This implies that the labor share in the model equals 59% — the average labor share for the period 1995–2012 according to the OECD database. The capital depreciation rate $\delta = 0.072$ is calibrated to match the average ratio of investment to output, which is approximately 24% in Poland. We use a fairly standard value for the discount rate of $\beta = 0.98$, which implies a net interest rate $R - \delta$ of 5.3% in the model. In our baseline calibration, we assume the coefficient of relative risk aversion $\gamma = 2$.

To pin down the unemployment replacement rate, we use data provided by van Vliet and Caminada (2012). The average net replacement rate in the period 1996–2013 equals 0.3 for a single worker and 0.35 for a one-earner couple with two children. However, these numbers are calculated for a six-month unemployment spell. In Poland, the average unemployment duration exceeds one year, but in most cases, the unemployment benefit is paid only for six months. As a result, we set $\theta_u = 0.2$, which is approximately half the estimates mentioned above. According to the OECD database, the net pension replacement rates in Poland are close to $\theta_r = 0.6$.

The maximum debt level in the model $\underline{k}(s)$ depends on skills. We assume that it is proportional to the average net income from work for agents with a given skill level:

$$\underline{k}(s) = \zeta(1-\bar{\tau})\xi_s l \bar{W}, \tag{11}$$

Table 1
Baseline calibration of the model.

Description	Parameter	Value
Capital share in the production function	α	0.41
Capital depreciation rate	δ	0.072
Discount rate	β	0.98
Disk aversion	γ	2
Std. dev. of aggregate shock	ΔZ	0.014
Unemployment replacement rate	θ_u	0.2
Pension replacement rate	θ_r	0.6
Relative debt level	ζ	0.083
Share of low-, medium- and high-skilled workers	ω	{0.111; 0.680; 0.209}
Individual labor supply	l	1

Table 2
Calibration of the individual labor efficiency factors.

Skill level				Age				
	Low	Medium	High		20–29	30–39	40–49	50–59
ξ_s	0.845	1	1.765	ξ_a	0.748	1	0.984	0.941

where ζ is the relative debt level, and $\bar{\tau}$ and $\bar{W}$ represent the average tax rate and wage, respectively.[2] We set a rather tight limit $\zeta = 0.083$. As a result, approximately 3% of agents, primarily the youngest and oldest, hold negative wealth in the model, which matches the number reported by the National Bank of Poland (NBP, 2015).

Regarding the skill level, we have three groups of agents in the model: low-, medium- and high-skilled, broadly corresponding to people with primary, secondary and tertiary education levels, respectively. Their average shares in economically active population in the period 1998–2014 equal 11.1, 68 and 20.9%, respectively, according to the statistical yearbooks of labor market in Poland. The individual labor supply l is set to 1. It is a redundant parameter and has no effect on the results.

The individual efficiency factors ξ_s and ξ_a shown in Table 2 are calibrated to match the observed differences in wages according to the Structure of Earnings Survey conducted by Eurostat in 2010. The data show that the earnings of low-skilled (primary-educated) workers are approximately 15 percent lower compared to medium-skilled (secondary-educated) workers. The earnings for high-skilled (tertiary-educated) workers are approximately 75 percent higher. Similarly, it is documented that young people (20–29 years) in Poland earn approximately 25 percent lower wages than the 30-39 years cohort. The earnings decrease slightly for older workers. The exact productivity factors of agents in a certain age a are interpolated using linear splines starting from 0.5 for the first cohort. This allows us to better capture the gradual increase in earnings of young workers.

3.2. Parameters of the stochastic structure

To calibrate the Markov chain for the aggregate productivity, we calculate log-deviations from the HP-filtered trend of yearly GDP covering the period 1995–2013. We consider artificial GDP data that consist of consumption and investment series only. As a result, we end up with symmetric business cycles where each phase lasts 3.75 years on average with productivity shock values of $Z_b = 0.986$ and $Z_g = 1.014$. The aggregate productivity transition matrix equals:

$$P_Z = \begin{bmatrix} 0.733 & 0.267 \\ 0.267 & 0.733 \end{bmatrix}. \tag{12}$$

The transition probabilities P_ε are pinned down to match the average level and duration of unemployment in Poland during booms and recessions for workers of different skill levels and age. We use the yearly data from the Labour Force Survey for the years 1997–2013. Because the data on the mean level of unemployment are grouped into five-year bins, the values for each year are linearly interpolated. The data also show that the unemployment duration in Poland is virtually equal for all education levels, and they distinguish only two age groups: 20–39 and 40–59. A similar assumption

[2] In fact, we do not use the exact average wage, as we have to set the debt limit $\underline{k}(s)$ at the beginning of the computations. Therefore, after some initial simulations, we make a guess on the aggregate capital stock K and use formula (5) to calculate $\bar{W}$. However, we do not have to employ the similar procedure for pinning down $\bar{\tau}$, as it solely depends on the labor supply, which is completely exogenous in the model.

on the constant unemployment duration across skill levels is used, for example, by Mukoyama and Şahin (2006).

After joining the European Union in 2004, many Polish citizens, especially young who were unable to find a satisfactory job in Poland, left the domestic labor market and moved abroad. As a result, our estimates of the unemployment rates are likely to be underestimated. Unfortunately, because of the lack of reliable data it is impossible to establish the exact business cycle properties of the international workforce flows, which are crucial for assessing the importance of migration for our estimates of the gains from eliminating business cycle fluctuations on labor market. Therefore, we have to neglect its impact.

The labor market transition probabilities are calculated in the following way (Table 3). Let $\bar{u}(s,a,Z)$ denote the mean unemployment level for a cohort a with the skill level s and the aggregate state Z calculated from the data, and let $u_l(a,Z)$ be the analogous average unemployment duration. To match the unemployment duration, the probability of finding a job $p_{\varepsilon,ue}$ is a reciprocal of the unemployment duration. If there is a switch between the aggregate states of the economy, we set this probability as a reciprocal of the mean value of the unemployment duration for boom and recession:

$$p_{\varepsilon,ue}(s,a,Z,Z\prime) = \begin{cases} u_l^{-1}(a,Z) & \text{if } Z = Z\prime \\ [0.5\,(u_l(a,Z) + u_l(a,Z\prime))]^{-1} & \text{if } Z \neq Z\prime \end{cases}. \quad (13)$$

Moreover, we assume that the aggregate unemployment level can take only two values that depend on the state of the economy. To ensure that this requirement is satisfied and that the unemployment rates in the model always match the data, we set the probability of losing a job as follows:

$$p_{\varepsilon,eu}(s,a,Z,Z\prime) = \frac{\bar{u}(s,a+1,Z\prime) - \bar{u}(s,a,Z)p_{\varepsilon,ue}(s,a,Z,Z\prime)}{1-\bar{u}(s,a,Z)}. \quad (14)$$

Formula (14) guarantees that if the current unemployment rate equals $\bar{u}(s,a,Z)$, then, in the next period, it switches to $\bar{u}(s,a+1,Z\prime)$.

Finally, the survival rates $q(a,a+1)$ are taken from Polish unisex lifetables from 2012.

4. Calculating the welfare gain from eliminating business cycles

To calculate the welfare gain from eliminating business cycles, we generally follow the definition proposed by Lucas (1987), with some minor modifications (see Storesletten et al., 2001). The measure is based on a comparison of the value functions in two economies: with and without aggregate fluctuations. To approximate the value functions, we employ the standard approximate aggregation algorithm proposed by Krusell and Smith (1998). The algorithm has already been used for solving overlapping generations models by Storesletten et al. (2001) and Heer and Maussner (2009), to name but a few. Details of the computations are provided in the appendix.

4.1. Definition of the welfare gain

In his seminal paper, Lucas (1987) defined the welfare gain as a percentage increase of consumption in the economy with business cycles needed to achieve the same lifetime utility as in an economy without business cycles. We use a similar concept with slight adjustments to the finite lifetime environment.

We define the lifetime welfare gain from eliminating business cycles as a percentage compensation in lifetime consumption of *an average newborn agent* in the economy with aggregate fluctuations needed to achieve the same lifetime utility as the mean lifetime utility of an average newborn agent *during the transition period* from the economy with aggregate fluctuations to the economy without them. The transition begins in the first period, when the aggregate shock disappears, and lasts until the aggregates reach a new steady state.

Therefore, we first calculate the gains λ_t for every period $t \in [1,T]$ of the transition. In what follows, we still omit the time subscripts for most variables. We use them only in a few cases to emphasize that the gain varies during the transition. Let $\Gamma(k,a,\varepsilon,s \mid K,Z)$ denote the conditional density of an agent's characteristics given K and Z, and let $\Gamma_{K,Z}(K,Z)$ be the unconditional density of the characteristics of the economy with aggregate fluctuations. Moreover, $V_t(k,a,\varepsilon,s,H_t(K);\lambda_t)$ and $\Gamma_t(k,a,\varepsilon,s \mid H_t(K))$ are the value function and the density of the individual characteristics in period t of the transition, respectively, and $K_t = H_t(K)$ represents the aggregate wealth in period t given K at the beginning of the transition. Finally, we remind that $V(k,a,\varepsilon,s,K,Z)$ stands for the value function in the economy with business cycles. Then, the gain solves the following equation:

$$\mathbb{E}V(k,20,\varepsilon,s,K,Z) = \mathbb{E}V_t(k,20,\varepsilon,s,H_t(K);\lambda_t), \quad (15)$$

where:

$$\mathbb{E}V(k,20,\varepsilon,s,K,Z) = \int\cdots\int V(k,20,\varepsilon,s,K,Z)\Gamma(k,20,\varepsilon,s \mid K,Z)\Gamma_{K,Z}(K,Z)dk\,d\varepsilon\,ds\,dK\,dZ, \quad (16)$$

$$\mathbb{E}V_t(k,20,\varepsilon,s,H_t(K);\lambda_t) = \int\cdots\int V_t(k,20,\varepsilon,s,H_t(K);\lambda_t)\Gamma_t \times(k,20,\varepsilon,s \mid H_t(K))\Gamma_{K,Z}(K,Z)dk\,d\varepsilon\,ds\,dK\,dZ, \quad (17)$$

Table 3
Labor market characteristics in Poland.

Skills	Phase	Age							
		20–24	25–29	30–34	35–39	40–44	45–49	50–54	55-59
A. Unemployment rates [%]									
Low	Rec.	46.1	37.3	29.1	26.5	23.5	22.2	19.8	13.5
	Boom	35.3	24.9	22.6	18.9	17.8	15.4	12.1	9.5
Med.	Rec.	32.4	18.5	14.4	13.2	12.8	12.5	12.0	11.1
	Boom	23.0	12.9	10.5	9.1	8.6	8.3	7.9	8.1
High	Rec.	25.9	11.3	4.3	2.8	2.4	2.7	3.4	4.2
	Boom	19.2	8.1	2.8	2.2	1.6	2.1	2.8	3.2
B. Unemployment durations [months]									
All	Rec.	15.1	15.1	15.1	15.1	19.8	19.8	19.8	19.8
	Boom	13.0	13.0	13.0	13.0	17.4	17.4	17.4	17.4

$$V_t(k,a,\varepsilon,s,H_t(K);\lambda_t) = \max_{c,k\prime}\{U((1+\lambda_t)c) + \beta q_{a,a+1}\mathbb{E}[V_t(k\prime,a+1,\varepsilon',s,H_{t+1}(K);\lambda_t) \mid \varepsilon]\} \quad \text{s.t. } (8)\text{–}(9). \tag{18}$$

Given the CRRA utility function (10), the gain can be explicitly calculated as:

$$\lambda_t = 100\cdot\left[\left(\frac{\mathbb{E}V_t(k,20,\varepsilon,s,H_t(K);0)}{\mathbb{E}V(k,20,\varepsilon,s,K,Z)}\right)^{\frac{1}{1-\gamma}} - 1\right]. \tag{19}$$

We assume that the gain is held equal across all agents. The gain for the whole transition period λ, which is the main measure in our paper, is simply calculated as the average across the λ_t:

$$\lambda = \frac{1}{T}\sum_{t=1}^{T}\lambda_t. \tag{20}$$

In the next section we also report values at the beginning λ_1 and at the end λ_T of the transition to assess the general equilibrium effect for the welfare gains.

Evaluating the expectations of the value functions, we integrate over individual capital stock, labor market status, skill level and aggregate characteristics. This is also the case for the value function on the transition path, because the aggregate capital K_t depends on the initial aggregate wealth before eventually converging to some steady state value. We do not integrate over age because we compare the value functions only for newborn agents. We also consider the welfare gains for agents with different skill levels. Consequently, we do not integrate the value functions over the skill level.

Our definition of the welfare gain from eliminating business cycles resembles the proposal of Storesletten et al. (2001). They also consider the average gain on the transition path. However, there is at least one important difference. The cited authors calculate the gains for every cohort separately and then average them. We report only the gain for newborn agents, as we find this measure closer to the lifetime gain usually used in infinite horizon frameworks. In other words, we focus on the individual's lifetime gain instead of the average economy-wide gain.

We also calculate momentary welfare gains for cohorts, where we simply compare average instantaneous utilities for a given cohort. Thus, we apply formulas (19) and (20), but we replace the value functions with the instantaneous utilities for particular cohorts.

4.2. Economy without business cycles

For the hypothetical stabilized economy, we have to distinguish the effects of eliminating business cycles on the aggregate and idiosyncratic risks.

For the aggregate risk, the problem is rather straightforward. Eliminating business cycles means shutting off the aggregate productivity shock. Thus, in the economy without business cycles, we set $Z = 1$.

For the idiosyncratic risk, the problem is less obvious. A wide range of proposals has been discussed in the literature (see Krusell and Smith, 1999, for a more detailed discussion). In the paper, we consider two schemes: direct stabilization of unemployment rates (Imrohoroğlu, 1989; Reiter, 2012) and direct stabilization of labor market transition probabilities (Hairault et al., 2010; Iliopulos et al., 2014; Jung and Kuester, 2011; Petrosky-Nadeau and Zhang, 2013).

The stabilization of unemployment rates simply postulates that unemployment rates as well as their durations in the economy without business cycles are constant and are set as averages of the corresponding characteristics across booms and recessions. To apply this method in our paper, we calculate the mean unemployment rates and durations for different cohorts and skill-level groups and then build the two-state transition matrices using formulas (13) and (14).

According to the second method, the transition matrices in the economy without business cycles equal the weighted averages of the corresponding matrices for different phases of the business cycles. In other words, we revert the ordering of the operations from the previous approach. First, we average the transition matrices. Then, from the matrices, we can then infer the stabilized unemployment rates and durations, which are usually slightly lower than in the previous case. As a result, the welfare gain from eliminating business cycles is usually higher in this setup. The discussed method is firmly grounded in the search and matching theory of the labor market. Indeed, in a study using a standard search and matching business cycle model, Hairault et al. (2010) showed that the transition probabilities are leveled off by eliminating the aggregate risk levels.

At this point, we should also mention the third popular approach — the integration principle. Here, the aggregate shock is integrated out of the risk process faced by individuals. However, for our model, the procedure is computationally cumbersome. This is mainly because unemployment rates in the stabilized economy are not immediately constant but need some time to converge to a steady state. Therefore, one has to add unemployment rates as the new state variable, which makes the model far more computationally complex. For this reason, we do not use this approach in the paper.

5. Results

5.1. Baseline calibration

5.1.1. Lifetime gains

In Table 4, we present the lifetime gains for our baseline calibration. The absolute gains are always reported in the paper as percentages. As mentioned earlier, we consider two stabilization schemes: the stabilized transition probabilities and the stabilized unemployment rates and durations. For each scheme, we separately report the gains for all newborn agents regardless of skill level as well as for low-, medium- and high-skilled agents.

In the first row, we present λ— the average gains for all transition periods. First, one can spot the considerable differences between the stabilization schemes. Under the stabilized probabilities, the gain for all agents equals 0.171% of the lifetime consumption. On the other hand, for the stabilized unemployment scheme, the gain is only

Table 4
Lifetime gains for the baseline calibration.

	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
λ	0.171	0.151	0.174	0.182	0.047	0.045	0.045	0.056
λ_1	0.171	0.150	0.173	0.182	0.054	0.053	0.053	0.063
λ_T	0.168	0.148	0.170	0.179	0.042	0.040	0.041	0.051
λ_{un}	0.163	0.149	0.165	0.171	0.026	0.036	0.026	0.013
λ_{em}	0.158	0.147	0.162	0.151	0.040	0.046	0.039	0.036

The gains are reported as percentages; λ — average lifetime gain; λ_1 — gain at the beginning of the transition; λ_T — gain at the end of the transition; λ_{un} — average gain for a newborn unemployed agent; λ_{em} — average gain for a newborn employed agent; Low, Med., High refer to the skill levels.

0.047%, which means that it is approximately four times smaller. In both cases, there are also differences across skill levels, especially between low- and high-skilled agents (0.151 and 0.182% under the stabilized probabilities scheme and 0.045 and 0.056% for the alternative stabilization policy).

The next two rows, containing the gains at the beginning (λ_1) and the end (λ_T) of the transition period, illustrate the general equilibrium effect. The effect is relatively negligible for the stabilized probabilities case but is sizeable for the stabilized unemployment scheme. Nonetheless, in both cases, the gains at the end of the transition are always lower than at the beginning. In the economy with business cycles, the aggregate capital stock is higher compared to the stabilized environment, as are wages. Because the labor-related income for an average agent exceeds the interest, the gains are partially offset by the increase in wages.

Finally, the last two rows show the gains for unemployed and employed newborn agents. Interestingly, the relative position of these two groups is different in the two stabilization schemes. Under the stabilized probabilities, the gains are higher for agents who enter the labor market as unemployed, whereas for the stabilized unemployment, the opposite is true. At first glance, these results may seem somewhat counterintuitive but one has to keep in mind that in calculating the gains, we compare agents of the same type. Thus, the results simply show that newborn unemployed agents benefit more from stabilizing the transition probabilities compared to employed agents, whereas the unemployment stabilization policy favors agents who enter the labor market as employed.

5.1.2. Momentary gains

Fig. 1 shows the age profiles of the average momentary gains for the transition period. In all cases, the gains are definitely highest for the youngest agents. They rapidly decrease with age, reaching the minima for 35–45-year-olds. Then, for the older cohorts, the gains either remain low or grow slightly. As far as the stabilized probabilities scheme is concerned, the gains are initially highest for high-skilled agents but then, as they become older, the gains drop considerably and remain lower than for the other skill groups. Similar effect is observed for low-skilled agents under the stabilized unemployment scheme.

The gain profiles are noticeably unstable for the young cohorts. This is the effect of the tight debt limit which is binding for many young agents and results in slight differences in consumption profiles in the economy with and without aggregate fluctuations. The differences translate into the variations in the early part of the gain profiles. As shown in Section 5.3.2, the profiles are smooth if the debt limit is looser.

To better capture the disparities in the momentary gains between cohorts, we calculate the average momentary gains for the key cohort bins and relate them to the average momentary gains for all generations, where in all cases, the cohorts sizes are used as weights. These results for age bins 20–24, 25–29 and 90–99 are given in Table 5.

The second row contains the average momentary gains for all cohorts. As expected, they are generally close to the average lifetime gains λ for newborn agents. Looking at the relative measures, we can see that the gains for the young cohorts are approximately two to four times higher than the average under the stabilized probabilities scheme and four to ten times higher for the stabilized unemployment case. The latter result is mainly due to the low average gains for all cohorts, as, in absolute terms, the gains for the young cohorts under the stabilized unemployment are still lower than under the alternative countercyclical policy. The gains are often higher for the 25–29 cohort compared to the 20–24 one because the latter is better insured against unemployment risk by the relatively higher unemployment benefits. These results show that the decline on consumption caused by business cycles is 2–10 times higher for young agents than the average for the whole population. Finally, the relative gains for the oldest agents are always close to 0.5.

5.1.3. General equilibrium effect for the momentary gains

Finally, we also investigate the general equilibrium effect for the momentary gains, which is depicted in Fig. 2. The upper graphs show the age profiles of the gains at the beginning of the transition, whereas the lower graphs present the profiles for the last period of the transition. Under both stabilization schemes, the general equilibrium effect lowers the gains considerably for mid-age cohorts and increases them for older agents. The former impact is especially noticeable under the stabilized unemployment scheme, where the gains for 40-year-olds drop from 0.02% at the beginning of the transition to −0.04% at the end. On the other hand, the latter effect dominates in the stabilized probabilities case where the gains increase fivefold for the 81–90 cohort.

5.2. Impact of the life-cycle heterogeneity of unemployment risk

To assess the impact of the life-cycle heterogeneity of unemployment risk, we study the model where the risk is constant across cohorts. More precisely, we calibrate the labor market transition probabilities to match the averages of the unemployment levels and the durations for all cohorts. The results of the exercise are presented in Table 6.

The lifetime gains decrease considerably, especially in the stabilized unemployment case (from 0.171 to 0.149% and from 0.047 to 0.028%). Currently, business cycles are most costly for low-skilled agents and far less severe for high-skilled ones, despite the flat unemployment benefits. Some changes are observed for the relative momentary gains too. When the life-cycle heterogeneity of unemployment risk is removed, the relative gains for the youngest cohorts drop in most cases: up to 30 percentage points under the stabilized probabilities scheme and up to 360 percentage points under the alternative. Nonetheless, the gains generally remain high. These results show that the life-cycle heterogeneity of unemployment risk significantly increases the lifetime gains from eliminating business

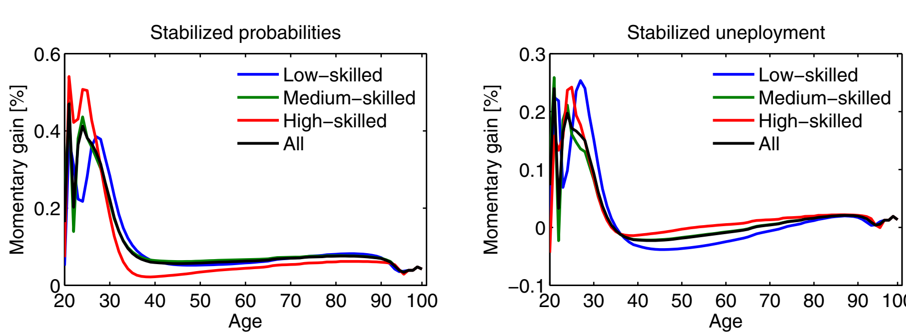

Fig. 1. Momentary gains for the baseline calibration.

Table 5
Gains for the baseline calibration.

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
λ	0.171	0.151	0.174	0.182	0.047	0.045	0.045	0.056
$\bar{\lambda}_{20\text{–}99}$	0.117	0.115	0.120	0.105	0.024	0.022	0.023	0.031
$\bar{\lambda}_{20\text{–}24,rel}$	2.7	2.1	2.7	3.8	5.9	5.5	6.5	4.1
$\bar{\lambda}_{25\text{–}29,rel}$	2.9	3.0	2.7	3.5	6.4	9.9	5.9	5.6
$\bar{\lambda}_{90\text{–}99,rel}$	0.5	0.5	0.5	0.5	0.6	0.5	0.6	0.5

The absolute gains are reported as percentages; Low, Med., High refer to the skill levels; $\bar{\lambda}_i$ denotes the average momentary gain for the age group i; $\bar{\lambda}_{i,rel} = \bar{\lambda}_i/\bar{\lambda}_{20\text{–}99}$ represents the relative momentary gains.

cycles but is by no means crucial for explaining the severity of business cycle fluctuations for young people.

To better capture the impact of youth unemployment risk, we conduct another exercise in which we stabilize unemployment risk for young agents only and calculate the welfare gains from removing business cycle fluctuations of the risk in the remaining cohorts. The results are presented in Table 7. We stabilize labor market fluctuations for the 20–24 and 20–29 cohorts in the first and second panels, respectively. We observe that the average lifetime gains drop substantially: in panel A, they decrease from 0.171 to 0.109% under the stabilized probabilities policy and from 0.047 to 0.015% under the stabilized unemployment scheme. For high-skilled agents, the effect is even more striking, as the lifetime gains are cut by 50 and 75%, respectively. Stabilizing labor market business cycles for another five-year bin reduce the gains further in quite similar proportions, as documented in panel B. As a result, the drop in the gains reaches even 90% for high-skilled agents under the stabilized unemployment scheme.

5.3. *Alternative calibrations*

In this subsection, we analyze how changes in the key parameters of the model affect the results. Subsequently, we consider the models with a higher risk aversion coefficient $\gamma = 4$, higher debt limit and newborn agents who enter the labor market with some inherited wealth. The results for these parametrizations are collected in Table 8.

5.3.1. *Higher risk aversion*

It is well known that rising risk aversion increases the lifetime welfare gains from eliminating business cycles. This is also the case for our model, where the lifetime gains soar from 0.171 to 0.437% for the stabilized probabilities policy and from 0.047 to 0.27% for the alternative scheme. The relative momentary gains do not change much. Under the stabilized probabilities scheme, they rise by several dozens of percentage points whereas they are usually slightly lower under the alternative scheme.

5.3.2. *Higher debt limit*

Setting the much looser debt limit $\zeta = 1$ leads to the considerably lower gains that are equal to 0.136 and 0.022% for the two studied schemes. In the latter case, the relative momentary gain is very high: for low-skilled agents almost 50 times higher than the average for all cohorts. Contrary to the previously studied cases, the gains for the oldest cohorts exceed the average gains as well. These effects result from the low levels of the average gains which barely exceed 0 as the absolute momentary gains for most cohorts are negative. In other words, the costs of business cycles are borne almost exclusively near the beginning and near the end of life.

As already mentioned, the gain profiles are no longer unstable as observed in the baseline parametrization. The smooth profiles are depicted on Fig. 3.

5.3.3. *Agents enter the labor market with inherited wealth*

In this exercise, newborn agents start with wealth inherited from agents of the same skill-type who die in a previous period. However,

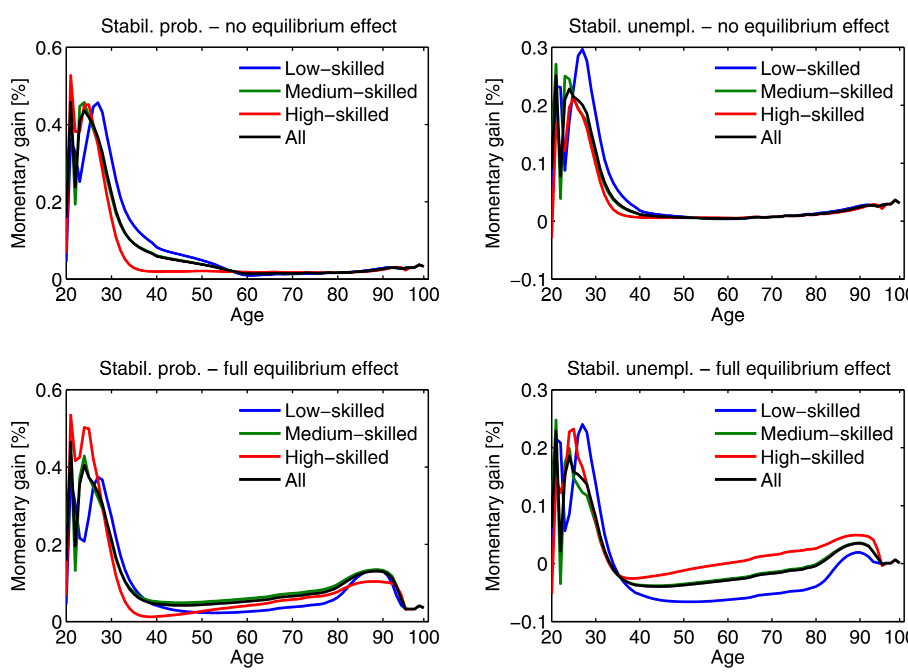

Fig. 2. General equilibrium effect for the momentary gain profiles.

Table 6
Gains under the homogeneous life-cycle unemployment risk.

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
λ	0.149	0.157	0.153	0.113	0.028	0.029	0.029	0.023
$\bar{\lambda}_{20-99}$	0.114	0.122	0.117	0.086	0.018	0.017	0.018	0.017
$\bar{\lambda}_{20-24,rel}$	2.2	2.1	2.2	2.4	5.8	6.5	5.9	3.8
$\bar{\lambda}_{25-29,rel}$	2.2	2.2	2.2	2.3	5.3	6.2	5.4	4.0
$\bar{\lambda}_{90-99,rel}$	0.5	0.4	0.5	0.6	0.7	0.6	0.7	0.8

The absolute gains are reported as percentages; Low, Med., High refer to the skill levels; $\bar{\lambda}_i$ denotes the average momentary gain for the age group i; $\bar{\lambda}_{i,rel} = \bar{\lambda}_i/\bar{\lambda}_{20-99}$ represents the relative momentary gains.

we continue to assume that leaving bequests provides no utility for testators. As a result, a vast majority of agents enter the labor market with some positive wealth, which helps to insure themselves against unemployment risk during the initial periods of their job market careers. As shown in panel C, both the lifetime gains and the absolute momentary gains for young and old cohorts decline, especially under the stabilized unemployment scheme. Nonetheless, in relative terms, the young are still two to three times worse off than the whole population under the stabilized probabilities scheme, whereas under the alternative variant they suffer even more than under the baseline calibration.

5.4. Proportional unemployment benefits

In this subsection, we study the consequences of replacing the flat unemployment benefits with the proportional benefits. Currently, we assume that the benefit is related to the average labor income for a given skill level group with the same replacement rate as in the baseline calibration (Table 9).

The resulting average lifetime gains are only slightly higher than in the baseline case, but their distribution across skill level groups changes. The gains for medium-skilled agents rise, whereas they decline for the two remaining groups. The changes for high- and medium-skilled groups reflect the changes in the absolute benefits, which increase for the former group and decrease for the latter. However, such relationship is not observed for low-skilled workers. The gains for this group are much smaller than in the baseline case and even negative under the stabilized unemployment scheme despite the decline in the unemployment benefits. This surprising result is the effect of the very low income of debt-constraint newborn unemployed agents. Their consumption turns out to be considerably higher in the economy with aggregate risk than in a world without business cycles and this slashes the gains.

For the calibration with the proportional unemployment benefits, we also remove the life-cycle heterogeneity of the unemployment risk, as in Section 5.2. The results collected in panel B are similar to those for the baseline calibration. The lifetime gains drop considerably, but the relative momentary gains remain quite high.

5.5. New pension scheme

Finally, we conduct an experiment that may provide insights into the behavior of the welfare gains in the future. We analyze the impact of the pension reforms that have recently been implemented in Poland as well as in many other countries as a response to the increase of life expectancy coupled with lower fertility rates. As a remedy to these trends, the statutory retirement age in Poland was increased from 60 years for women and 65 years for men to 67 years for both sexes. Despite this action, the pension replacement rates are expected to drop by half.

This exercise is intended not to assess the pension reform itself. Instead, it should be treated as another robustness check for the results presented in the previous sections when the new values of the parameters related to pension scheme are considered.

In our setup, the reform is represented by an alternation of three parameters. First, we proportionally reduce all death probabilities by half. As a result, life expectancy in the model rises by 7.5 years, which is close to the central demographic projection for Poland for 2050 year (CSO, 2014). Second, we increase the retirement age in the model from 60 to 65 years, as we continue to assume that the average actual retirement age will be lower than the statutory one. Finally, we cut the pension net replacement rate from 60 to 30 percent according to simulations of the Social Security Office in Poland (Kwiecińska, 2011).

The results of these reforms for the welfare gains are shown in Table 10. At the beginning, we consider all changes taken together (panel A). Then, we study the effects of every modification separately (panels B–D). First, we can see that under the new pension scheme, the lifetime gains decline moderately for all agents, from 0.171 to 0.149% for the stabilized probabilities and from 0.047 to 0.035% for the alternative policy. The reform is almost equally beneficial for all skill-level groups. This result is mainly driven by the decline

Table 7
Gains with stabilized business cycles in labor market for young cohorts.

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
A: Business cycles on labor market stabilized for cohorts 20–24								
λ	0.109	0.117	0.112	0.085	0.015	0.020	0.014	0.014
$\bar{\lambda}_{20-99}$	0.091	0.100	0.094	0.066	0.011	0.012	0.010	0.013
$\bar{\lambda}_{20-24,rel}$	1.0	0.8	1.1	1.2	2.7	1.6	3.4	0.6
$\bar{\lambda}_{25-29,rel}$	3.2	3.4	3.0	4.3	10.5	15.3	9.9	7.9
$\bar{\lambda}_{90-99,rel}$	0.5	0.5	0.5	0.7	1.0	0.7	1.0	0.9
B: Business cycles on labor market stabilized for cohorts 20–29								
λ	0.069	0.085	0.070	0.041	0.005	0.007	0.005	0.005
$\bar{\lambda}_{20-99}$	0.057	0.072	0.057	0.034	0.006	0.006	0.006	0.007
$\bar{\lambda}_{20-24,rel}$	0.6	0.5	0.6	0.9	2.3	2.6	2.3	1.7
$\bar{\lambda}_{25-29,rel}$	1.6	1.5	1.6	1.9	4.4	5.8	4.4	2.8
$\bar{\lambda}_{90-99,rel}$	0.4	0.3	0.4	0.7	1.0	0.9	1.0	0.9

The absolute gains are reported as percentages; Low, Med., High refer to the skill levels; $\bar{\lambda}_i$ denotes the average momentary gain for the age group i; $\bar{\lambda}_{i,rel} = \bar{\lambda}_i/\bar{\lambda}_{20-99}$ represents the relative momentary gains.

Table 8
Gains for the alternative calibrations

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
A: Higher risk aversion $\gamma = 4$								
λ	0.437	0.342	0.446	0.670	0.270	0.226	0.274	0.385
$\bar{\lambda}_{20\text{–}99}$	0.189	0.173	0.191	0.215	0.081	0.073	0.080	0.118
$\bar{\lambda}_{20\text{–}24,rel}$	2.8	2.0	3.0	4.1	4.3	3.2	4.5	4.3
$\bar{\lambda}_{25\text{–}29,rel}$	4.0	4.1	3.9	5.0	6.3	7.6	6.0	6.0
$\bar{\lambda}_{90\text{–}99,rel}$	0.6	0.6	0.7	0.6	0.5	0.2	0.6	0.6
B: Higher debt limit $\zeta = 1$								
λ	0.136	0.126	0.140	0.131	0.022	0.024	0.021	0.030
$\bar{\lambda}_{20\text{–}99}$	0.101	0.095	0.105	0.084	0.009	0.004	0.008	0.017
$\bar{\lambda}_{20\text{–}24,rel}$	3.0	3.0	2.8	4.0	15.3	45.1	15.4	6.8
$\bar{\lambda}_{25\text{–}29,rel}$	2.4	2.6	2.2	3.0	10.4	35.5	9.8	5.5
$\bar{\lambda}_{90\text{–}99,rel}$	0.9	0.9	0.9	0.9	3.0	4.8	3.2	2.0
C. Newborn agents start with inherited wealth								
λ	0.129	0.127	0.133	0.113	0.015	0.012	0.014	0.022
$\bar{\lambda}_{20\text{–}99}$	0.103	0.107	0.106	0.082	0.009	0.002	0.010	0.017
$\bar{\lambda}_{20\text{–}24,rel}$	2.3	2.1	2.2	3.0	8.7	48.3	8.3	4.2
$\bar{\lambda}_{25\text{–}29,rel}$	2.3	2.3	2.3	2.7	9.2	55.0	8.6	4.9
$\bar{\lambda}_{90\text{–}99,rel}$	0.4	0.4	0.4	0.5	0.7	2.3	0.6	0.5

The absolute gains are reported as percentages; Low, Med., High refer to the skill levels; $\bar{\lambda}_i$ denotes the average momentary gain for the age group i; $\bar{\lambda}_{i,rel} = \bar{\lambda}_i / \bar{\lambda}_{20\text{–}99}$ represents the relative momentary gains.

in the replacement rate, resulting in lower taxes, which facilitates insurance against unemployment risk. The higher life expectancy generally decreases the gains as well because it is associated with longer expected retired life without unemployment risk. However, this effect is partially neutralized by higher taxation. Although the increase in the retirement age affects the gains in the exactly opposite way, as it reduces both expected retired life and taxes, eventually it leads to a similar slight reduction of the lifetime gains regardless of skill level.

Despite the decline in the lifetime gains, the absolute momentary gains for young cohorts are often higher than those under the baseline pension setup. This result clearly suggests that in the new pension scheme, an even larger fraction of the business cycle costs will be borne by young generations, mainly due to the increase in the retirement age.

6. Conclusion

We study the welfare gains from eliminating business cycles in an OLG economy with the life-cycle heterogeneity of unemployment risk calibrated to Polish data. We find that most of the costs associated with business cycles are borne during the early stages of a labor market career and that this results from the interplay of the much higher unemployment risk faced by young agents and their limited abilities to self-insure against the risk. We also show that a countercyclical policy that aims to stabilize business cycle fluctuations on the labor market for young agents would reduce the costs considerably, not only from the perspective of these cohorts but, to the larger extent, from the lifetime point of view. The need for appropriate policy-actions will probably become stronger, as the recent pension reforms triggered by the demographic changes are likely to make young people even more exposed to business cycles, despite declining lifetime gains.

Our estimates of the gains should be regarded as a downward biased because we disregarded at least a few important factors that should increase either the lifetime or the relative momentary gains for young agents. For example, we did not distinguish between normal and long-term unemployment, as in Mukoyama and Şahin (2006) and Krusell et al. (2009). Indeed, Poland has a large share of long-term unemployed, although it is unclear to what extent this results from structural factors unrelated to business cycles. Similarly, we did not take into account observed dependencies between the labor market conditions youth face when looking for their first job and their subsequent job market careers (Burgess et al., 2003; Kahn, 2010). The possibility that being unemployed at the beginning of a job market career reduces employment probabilities and earnings

Table 9
Gains with proportional unemployment benefits.

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
A. Baseline calibration								
λ	0.197	0.097	0.224	0.164	0.052	−0.038	0.073	0.046
$\bar{\lambda}_{20\text{–}99}$	0.117	0.115	0.123	0.081	0.029	0.007	0.035	0.025
$\bar{\lambda}_{20\text{–}24,rel}$	3.2	−0.2	3.7	4.6	4.5	−30.8	6.0	5.0
$\bar{\lambda}_{25\text{–}29,rel}$	3.4	4.5	3.2	3.3	7.0	48.2	5.4	4.4
$\bar{\lambda}_{90\text{–}99,rel}$	0.2	0.2	0.2	0.3	0.4	1.5	0.4	0.6
B. Homogeneous life-cycle unemployment risk								
λ	0.161	0.108	0.181	0.102	0.027	−0.046	0.043	0.019
$\bar{\lambda}_{20\text{–}99}$	0.119	0.113	0.127	0.079	0.018	−0.005	0.023	0.015
$\bar{\lambda}_{20\text{–}24,rel}$	2.3	0.3	2.7	2.3	4.9	−0.173[a]	6.5	3.7
$\bar{\lambda}_{25\text{–}29,rel}$	2.3	2.9	2.2	2.0	5.8	0.133[a]	4.5	3.4
$\bar{\lambda}_{90\text{–}99,rel}$	0.5	0.5	0.4	0.7	0.7	0.010[a]	0.5	0.8

The absolute gains are reported as percentages; Low, Med., High refer to the skill levels; $\bar{\lambda}_i$ denotes the average momentary gain for the age group i; $\bar{\lambda}_{i,rel} = \bar{\lambda}_i / \bar{\lambda}_{20\text{–}99}$ represents the relative momentary gains.

[a] The absolute momentary gains for the skill-level groups $\bar{\lambda}_i$ are reported because of the negative average gains $\bar{\lambda}_{20\text{–}99}$.

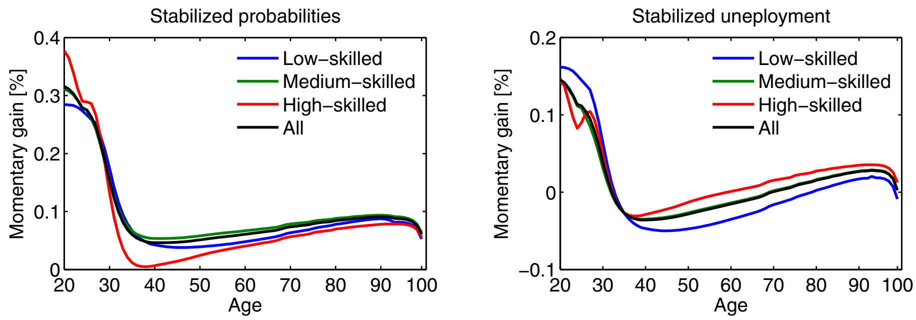

Fig. 3. Momentary gains for the higher debt limit calibration.

Table 10
Gains for the new pension scheme.

Group	Stabilized probabilities				Stabilized unemployment			
	All	Low	Med.	High	All	Low	Med.	High
A. New pension scheme								
λ	0.149	0.136	0.151	0.149	0.035	0.035	0.034	0.041
$\bar{\lambda}_{20-99}$	0.108	0.108	0.110	0.091	0.021	0.020	0.021	0.027
$\bar{\lambda}_{20-24,rel}$	2.6	2.2	2.5	3.9	5.4	7.1	5.4	3.5
$\bar{\lambda}_{25-29,rel}$	3.2	2.8	3.2	4.3	7.5	8.5	7.4	6.8
$\bar{\lambda}_{90-99,rel}$	0.6	0.7	0.6	0.6	0.8	0.7	0.8	0.7
B. New surviving probabilities only								
λ	0.169	0.152	0.171	0.177	0.042	0.041	0.041	0.051
$\bar{\lambda}_{20-99}$	0.104	0.112	0.104	0.094	0.022	0.020	0.021	0.030
$\bar{\lambda}_{20-24,rel}$	3.1	2.3	3.2	4.2	6.4	7.2	6.9	4.2
$\bar{\lambda}_{25-29,rel}$	3.5	3.3	3.4	4.1	7.3	10.7	6.9	6.2
$\bar{\lambda}_{90-99,rel}$	0.3	0.3	0.3	0.4	0.8	0.8	0.9	0.7
C. New retirement age only								
λ	0.167	0.147	0.169	0.178	0.047	0.046	0.045	0.055
$\bar{\lambda}_{20-99}$	0.114	0.111	0.116	0.100	0.024	0.023	0.023	0.032
$\bar{\lambda}_{20-24,rel}$	2.7	2.0	2.7	3.8	5.6	4.9	6.2	3.9
$\bar{\lambda}_{25-29,rel}$	2.9	3.1	2.7	3.6	6.3	9.8	5.8	5.5
$\bar{\lambda}_{90-99,rel}$	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6
D. New pension replacement rate only								
λ	0.155	0.139	0.157	0.159	0.038	0.036	0.037	0.046
$\bar{\lambda}_{20-99}$	0.102	0.110	0.102	0.090	0.023	0.022	0.023	0.031
$\bar{\lambda}_{20-24,rel}$	2.9	2.3	2.9	3.9	5.1	6.5	5.3	3.3
$\bar{\lambda}_{25-29,rel}$	3.4	3.0	3.4	4.1	6.5	7.8	6.3	5.9
$\bar{\lambda}_{90-99,rel}$	0.2	0.2	0.2	0.2	0.7	0.6	0.7	0.6

The absolute gains are reported as percentages; Low, Med., High refer to the skill levels; $\bar{\lambda}_i$ denotes the average momentary gain for the age group i; $\bar{\lambda}_{i,rel} = \bar{\lambda}_i/\bar{\lambda}_{20-99}$ represents the relative momentary gains.

for the next dozen or so years definitely increases the gains from eliminating business cycles for young agents. However, we leave these topics for further investigation.

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Appendix A. Derivation of the tax rates

Because both, government incomes and expenditures, are related to aggregate labor supply only, they solely depend on the aggregate state of the economy. Let $\Gamma(k, a, \varepsilon, s \mid K, Z)$ represent agents' density. Then, for each aggregate state Z, we define the aggregate tax income $G_w(Z)$ as well as the aggregate expenditures on unemployment benefits $G_u(Z)$ and pensions $G_r(Z)$:

$$G_w(Z) = \tau(Z) l W(Z) L_w(Z), \tag{A1}$$

$$G_u(Z) = [1 - \tau(Z)] l W(Z) \theta_u L_u(Z), \tag{A2}$$

$$G_r(Z) = [1 - \tau(Z)] l W(Z) \theta_r L_r(Z), \tag{A3}$$

$$L_w(Z) = \int \cdots \int \xi_s \xi_a \mathbb{I}(\varepsilon = e) \Gamma(k, a, \varepsilon, s \mid K, Z) dk\, da\, d\varepsilon\, ds, \tag{A4}$$

$$L_u(Z) = \int \cdots \int \bar{\xi} \mathbb{I}(\varepsilon = u) \Gamma(k, a, \varepsilon, s \mid K, Z) dk\, da\, d\varepsilon\, ds, \tag{A5}$$

$$L_r(Z) = \int \cdots \int \xi_s \xi_{59} \mathbb{I}(a \geq 60) \Gamma(k, a, \varepsilon, s \mid K, Z) dk\, da\, d\varepsilon\, ds. \tag{A6}$$

Clearly, for a budget to be balanced, we must have $G_w(Z) = G_u(Z) + G_r(Z)$, which implies that:

$$\tau(Z) = \frac{L_w(Z) + \theta_u L_u(Z) + \theta_r L_r(Z)}{\theta_u L_u(Z) + \theta_r L_r(Z)}. \tag{A7}$$

Appendix B. Computational details

To calculate the welfare gains, the following main steps are taken:

B.1 Solve the model without the aggregate risk to obtain a nonstochastic steady state for aggregate wealth K_{ss} and stationary density of wealth $\Gamma_{ss}(k,a,\varepsilon,s)$. These variables are then used as initial values in the subsequent steps of the procedure.
B.2 Solve the model with the aggregate risk using the Krusell–Smith approximate aggregation algorithm.
B.3 Simulate the model with aggregate risk to obtain the distribution of aggregate wealth and productivity shock $\Gamma_{K,Z}(K,Z)$. The density is approximated by a discrete density with 96 equidistant points for K and two points for Z. The simulated sample contains 100,500 periods, where first 500 observations are discarded.
B.4 Solve the model without the aggregate risk *during the transition* — when K moves from the average level for the model with the aggregate risk, given by $\iint K\Gamma_{K,Z}(K,Z)dK\,dZ$, towards K_{ss}. Despite the lack of aggregate shocks, K is not constant during the transition. Thus, we have to utilize the Krusell–Smith algorithm once more. This algorithm also gives a perceived law of motion for the aggregate wealth during the transition $H_t(K)$, given K as the initial value.
B.5 Given the policy functions from steps 2 and 4 as well as the perceived law of motion $H_t(K)$, calculate the value functions $V(k,20,\varepsilon,s,K,Z)$ and $V_t(k,a,\varepsilon,s,H_t(K);0)$.
Because k and K are continuous, we approximate the value functions for these variables on grids. For k, we use separate polynomial grids of order 5 with 300 points for every skill level group. The polynomial grid is much denser near the borrowing constraint, where the policy function is nonlinear. For a higher k, the policy function is nearly linear, and the grid can be sparser. Because we assume different maximum debt levels for agents with different skills, we also have to use different grids for them. In most calibrations, the upper bound for individual wealth is set to 50 for high-skilled agents. The bound is approximately 16 percent lower for medium-skilled agents and 21 percent lower for low-skilled agents. In the case of K, we utilize an equidistant grid with 10 points over an interval $[0.9K_{ss}1.1K_{ss}]$.
B.6 Integrate the value functions and calculate the welfare gains.

In what follows, we discuss the most important ingredients of the described procedure. A key computational difficulty is to solve an agent's decision problem, which is defined as:

$$V(k,99,\varepsilon,s,\Gamma,Z) = U(k+d), \tag{B1}$$

$$V(k,a,\varepsilon,s,\Gamma,Z) = \max_{c,k\prime}\{U(c) + \beta q_{a,a+1}\mathbb{E}[V(k',a+1,\varepsilon',s,\Gamma',Z') \mid \varepsilon,\Gamma,Z]\} \tag{B2}$$

$$\text{s.t.} \quad k' = (1-\delta+R)k + d - c, \tag{B3}$$

$$k\prime \geq \underline{k}(s), \tag{B4}$$

$$\Gamma\prime = \mathbb{H}(\Gamma,Z,Z'), \tag{B5}$$

where Γ represents the distribution of agents over (k,a,ε,s). The policy functions take the forms:

$$k\prime = k'(k,a,\varepsilon,s,\Gamma,Z), \quad c = c(k,a,\varepsilon,s,\Gamma,Z). \tag{B6}$$

Our algorithm uses three important concepts. First, the approximate aggregation property discussed by Krusell and Smith (1998) is utilized to make agents' expectations almost fully rational. Then, we use a backward iteration together with the Euler equation iteration algorithm proposed by Maliar et al. (2010) to find the explicit decision rules of agents from a given cohort. Finally, when simulating the model, we follow Heer and Maussner (2009, see pp. 544–545) and iterate on the agents' density over (k,a,ε,s) instead of simulating the decisions of large group of agents, as many studies have done. Below, we discuss these concepts in more detail. In general, the whole procedure is similar to that used by Heer and Maussner (2009, algorithm 10.2.1).

B.1. Krusell–Smith algorithm for the model with aggregate risk

A fully rational agent uses the distribution Γ together with its law of motion $\mathbb{H}$ to predict an aggregate capital level that determines future interest rates and wages. Because Γ is an infinite-dimensional object, it is impossible to approximate numerically the value function and the policy functions. However, as noted by Krusell and Smith (1998), in many cases, instead of the whole distribution, it is sufficient to use only its first few moments. In fact, in our case, taking into account only the first moment K provides satisfactory accuracy. Therefore, the problem can also be written in the form presented in the main body of the paper:

$$V(k,99,\varepsilon,s,K,Z) = U(k+d), \tag{B7}$$

$$V(k,a,\varepsilon,s,K,Z) = \max_{c,k\prime}\{U(c) + \beta q_{a,a+1}\mathbb{E}[V(k',a+1,\varepsilon',s,K',Z') \mid \varepsilon,K,Z]\} \tag{B8}$$

$$\text{s.t.} \quad k' = (1-\delta+R)k + d - c, \tag{B9}$$

$$k\prime \geq \underline{k}(s), \tag{B10}$$

$$K\prime = H(K, Z, Z'), \tag{B11}$$

where we use a simple loglinear law of motion for aggregate capital:

$$\ln K' = \begin{cases} b_{0b} + b_{1b} \ln K & \text{if } Z = Z_b \\ b_{0g} + b_{1g} \ln K & \text{if } Z = Z_g. \end{cases} \tag{B12}$$

Krusell and Smith (1998) proposed an intuitive iterative procedure to determine the coefficients $b = [b_{0b}, b_{1b}, b_{0g}, b_{1g}]$ that makes the perceived law of motion (B12) as rational as possible. The full rationality implies that the perceived law of motion coincides with the actual law of motion from the simulated model. The algorithm can be summarized as follows:

B.1.1 Set initial values of the coefficients $b^{(0)}$.
B.1.2 For a given $b^{(j)}$, find the decision rules $k' = k'(k, a, \varepsilon, s, K, Z)$ and $c = c(k, a, \varepsilon, s, K, Z)$ that solve the consumption–saving problem (B7)–(B12).
B.1.3 Given the decision rules, simulate the model for T periods and compute a time path for the mean aggregate capital.
B.1.4 Estimate the new autoregressive coefficients $b^{(j+1)}$ using ordinary least squares.
B.1.5 If $\|b^{(j+1)} - b^{(j)}\|_\infty < \nu_b$, then stop; otherwise update vector $b^{(j+1)} = \phi_b b^{(j+1)} + (1 - \phi_b) b^{(j)}$ and return to step 2.

In the baseline version of the model, we set $b^{(0)} = [1, 0, 1, 0]$, $\nu_b = 10^{-6}$ and $\phi_b = 0.75$. The model is simulated for $T = 4200$ periods, where the first 200 periods are used as a burn-in sample. The resultant law of motion is as follows:

$$\ln K' = \begin{cases} 0.0836 + 0.9522 \ln K & \text{if } Z = Z_b \\ 0.1053 + 0.9472 \ln K & \text{if } Z = Z_g. \end{cases} \tag{B13}$$

The law of motion fits the simulated data well. R^2 equals 0.999959 for bad aggregate state and 0.999960 for the good one.

B.2. Solving the individual decision problem by Euler equation iteration (step B.1.2)

To approximate the solution for an agent's decision problem given $b^{(j)}$, we use the backward iteration method, and, following Maliar et al. (2010), we employ the Euler equation iteration for a single cohort. As shown in the cited paper, the Euler equation is given by:

$$k' = (1 - \delta + R)k + d - \left[\eta + \beta q_{a,a+1} \mathbb{E}\left(\frac{1 - \delta + R\prime}{[(1 - \delta + R\prime)k\prime + d\prime - k\prime\prime]^\gamma}\right)\right]^{-\frac{1}{\gamma}}, \tag{B14}$$

where η is a Lagrange multiplier associated with the borrowing constraint (B10).

We look for the policy functions $k' = k'(k, a, \varepsilon, s, K, Z)$ and $c = c(k, a, \varepsilon, s, K, Z)$. The whole procedure can be summarized as follows:

B.2.1 Set the grids for individual k and aggregate wealth K.
B.2.2 Compute the decision rules for the last cohort:

$$k'(k, 99, \varepsilon, s, K, Z) = 0, \quad c(k, 99, \varepsilon, s, K, Z) = k + d. \tag{B15}$$

We assume that the last cohort leaves neither debt nor bequest. Therefore, it simply consumes its whole wealth.
B.2.3 For every cohort $a = 99 - i, i = 1, 2, \ldots, 79$ and skill level group s:
(a) Set the initial policy function $k'_0(k, a, \varepsilon, s, K, Z) = k\prime(k, a + 1, \varepsilon, s, K, Z)$, assuming that the initial policy function equals the policy for the next cohort.
(b) In Eq. (B14), set $\eta = 0$. Compute the new policy function k'_{i+1} on the predefined grid from the r.h.s. of Eq. (B14). The expectation term is based on the transition probabilities P_Z and P_ε. The next period interest rate R' and wage W' needed to calculate the future income d' are computed using the law of motion (B12) with coefficients $b^{(j)}$. To find a value of k'' on the grid k, we interpolate the next cohort policy $k'(k, a + 1, \varepsilon, s, K, Z)$ in points obtained from the current cohort policy $k'_i(k, a, \varepsilon, s, K, Z)$. We apply a piecewise cubic Hermite interpolation.
(c) If some elements of k'_{i+1} lie outside the capital grid domain, set them to the respective boundary values.
(d) If $\| k'_{i+1} - k'_i \|_\infty < \nu_k$, then move to the next step; otherwise, update the policy function $k'_{i+1} = \phi_k k'_{i+1} + (1 - \phi_k) k'_i$ and return to step (b).
(e) Compute the consumption policy from the budget constraint.

We use similar grids as in the value function approximation step. The only difference is that we now use only 150 nodes for the grids for k. Other parameters in the baseline version of the model are equal to: $\nu_k = 10^{-8}$ and $\phi_k = 0.25$.

B.3. Simulating individuals' wealth density (step B.1.3)

To simulate the dynamics of individuals' wealth density, we follow Heer and Maussner (2009, see pp. 544–545) and iterate on the agents' density over (k, a, ε).

We simulate only the aggregate productivity shock. Then, for every period and skill level group, we analytically compute individual capital density functions, taking advantage of the fact that with a discretized individual capital level, their dynamics are described by a Markov chain with transition probabilities depending on the aggregate capital as well as the current and future state of the economy. Because we do not allow for skill changes, we can build separate chains for each skill group. The states of the Markov chains are then defined by a triple (k, a, ε).[3] To describe precisely how the transition matrices $P_\Gamma(s, Z, Z')$ are constructed *given some fixed aggregate capital level K*, we introduce the following probabilities:

- $p_\Gamma^{(I,J)}(s, Z, Z')$ — probability that an agent moves from state $I = (i_k, i_a, i_\varepsilon)$ to $J = (j_k, j_a, j_\varepsilon)$;
- $p_k^{(i_k,j_k)}(a, \varepsilon, s, Z)$ — probability that an agent changes her capital stock from $k(i_k)$ to $k(j_k)$, where $i_k, j_k \in \{1, 2, \ldots, n_k\}$, and n_k represents the number of nodes in the individual capital grid:
  If $k(j_k) \le \tilde{k}\prime(k(i_k), a, \varepsilon, s, Z) \le k(j_k + 1)$, then:

$$p_k^{(i_k,j_k)}(a, \varepsilon, s, Z) = \frac{\tilde{k}\prime(k(i_k), a, \varepsilon, s, Z) - k(j_k)}{k(j_k + 1) - k(j_k)}, \tag{B16}$$

$$p_k^{(i_k,j_k+1)}(a, \varepsilon, s, Z) = 1 - p_k^{(i_k,j_k)}(a, \varepsilon, s, Z); \tag{B17}$$

  where $\tilde{k}'(k, a, \varepsilon, s, Z)$ denotes an interpolated policy function defined below.
- $p_Z^{(i_Z,j_Z)}$ — probability that the aggregate state of the economy switches from Z_{i_Z} to Z_{j_Z}, where $i_Z, j_Z \in \{b, g\}$. These probabilities are defined in the paper;
- $p_\varepsilon^{(i_\varepsilon,j_\varepsilon)}(a, s, Z, Z')$ – probability that the employment status of an agent moves from i_ε to j_ε, where $i_\varepsilon, j_\varepsilon \in \{u, e\}$. These probabilities are also defined in the paper.

Then:

$$p_\Gamma^{(I,J)}(s, Z, Z') = \begin{cases} (1 - q_{a(i_a),a(i_a)+1}) \cdot \bar{u}(20, s, Z(j_Z)) \cdot p_Z^{(i_Z,j_Z)} \cdot p_\varepsilon^{(i_\varepsilon,j_\varepsilon)} & \text{if } j_a = 20, \quad j_\varepsilon = u, j_k = j_0 \ \text{(new unempl.)} \\ (1 - q_{a(i_a),a(i_a)+1}) \cdot (1 - \bar{u}(20, s, Z(j_Z))) \cdot p_Z^{(i_Z,j_Z)} \cdot p_\varepsilon^{(i_\varepsilon,j_\varepsilon)} & \text{if } j_a = 20, \quad j_\varepsilon = e, j_k = j_0 \ \text{(new empl.)} \\ q_{a(i_a),a(i_a)+1} \cdot p_k^{(i_k,j_k)} \cdot p_Z^{(i_Z,j_Z)} \cdot p_\varepsilon^{(i_\varepsilon,j_\varepsilon)} & \text{if } j_a = i_a + 1, \quad j_a < 60 \ \text{(surv. work.)} \\ q_{a(i_a),a(i_a)+1} \cdot p_k^{(i_k,j_k)} \cdot p_Z^{(i_Z,j_Z)} & \text{if } j_a = i_a + 1, \quad j_a \ge 60 \ \text{(surv. retir.)} \\ 0 & \text{otherwise} \end{cases}$$

and j_0 represents a position of 0 in the grid for k whereas $\bar{u}(a, s, Z)$ is the average unemployment rate. Thus, agents who die return to the population as newborn, with no capital and either employed or unemployed. In these cases, the transition probabilities are given in the first two lines of formula (B18). Then, conditional on surviving to the next periods, their further path depends on the probabilities from the next two lines, which refer to agents of working age (third line) and upon retirement (fourth line).

The whole simulation procedure consists of the following steps:

B.1.3.1 Draw a sequence of T aggregate shocks;
B.1.3.2 Discretize k and set $t = 0$ and the initial density $\Gamma_t(k, a, \varepsilon, s) = \Gamma_{ss}(k, a, \varepsilon, s)$. Here, we use a nonstochastic stationary density (stationary density from the model without aggregate shocks);
B.1.3.3 Using the density Γ_t, calculate the aggregate capital K_t;
B.1.3.4 Create an interpolated policy function $\tilde{k}\prime(k, a, \varepsilon, s, Z)$ by interpolating the policy function $k'(k, a, \varepsilon, s, K, Z)$ at the new grid from step B.1.3.2 and the aggregate capital calculated in step B.1.3.3 It should be emphasized that, as already mentioned above, the grid for k in the wealth density simulation procedure does not necessarily coincide with the grid created for the policy iteration scheme;
B.1.3.5 For each skill level group, calculate the transition matrix $P_\Gamma(s, Z_t, Z_{t+1})$ using formulas (B16)–(B18);
B.1.3.6 Calculate the new density $\Gamma_{t+1}(k, a, \varepsilon, s)$. This density is a mixture of conditional densities with respect to the skill level groups, where the latter are simple products of current conditional densities and the respective transition matrices;
B.1.3.7 If $t = T$, stop; otherwise, set $t = t + 1$ and return to step B.1.3.3

Because the procedure is invoked repeatedly by the Krusell–Smith algorithm, we always use the same sequence of aggregate shocks. Otherwise, the procedure might not converge. With 300 nodes for k, the Markov chains representing the conditional densities of individual wealth have $300 \cdot (2 \cdot 40 + 40) = 48,000$ states each. As a result, the transition matrices P_Γ have approximately $2 \cdot 10^9$ entries. However, during computations they can easily be stored as sparse matrices, because the vast majority of their entries are zeros.

B.4. Solving the models without aggregate risk

B.4.1. Model for the transition

To solve the model without aggregate risk on the transition path, we still need to employ an algorithm similar to the Krusell–Smith procedure. This is because aggregate capital is not constant during the transition but slowly converges to the nonstochastic steady state. Therefore, one has to determine the perceived law of motion for aggregate capital that approximately coincides with rational expectations. The key

[3] Actually, the labor market status ε is redundant for retirees. In this case, we use only one value: $\varepsilon = u$.

deviations from the standard algorithm described in the previous subsections occur in the simulation step. First, the law of motion depends only on aggregate capital:[4]

$$K\prime = H_1(K) = \exp(b_0 + b_1 \ln K). \tag{B19}$$

Second, the simulations always start from the same point — the average level of aggregate capital in the model with aggregate risk — and finish when the aggregate capital stabilizes, provided that the minimum number of transition periods $T_{min} = 100$ is reached. Introducing the minimum duration of the transition in the Krusell–Smith procedure facilitates the convergence of the algorithm. Generally, for different values of b_0 and b_1 the duration of the transition could also differ. However, once the coefficients are eventually determined by the Krusell–Smith algorithm, we run a simulation of the transition path once more without the minimum number of periods restriction to establish the final duration of the transition. Obviously, one has to keep in mind that the policy functions during the transition no longer depend on aggregate shock.

We should stress that the transition starts from the average level of aggregate capital in the model with aggregate risk only to determine the law of motion $H_1(K)$. When we calculate the value functions, we take into account that it could start from other points as well. This is why we integrate over the density of aggregate capital. However, the duration of the transition is always kept constant, as determined by a single simulation after the Krusell–Smith algorithm.

B.4.2. Model with constant aggregate capital

When there are no aggregate shocks and the aggregate capital is kept at the nonstochastic steady state level, all other aggregates are also fixed. To solve such a model, standard procedures can be employed (Heer and Maussner, 2009). Basically, we only have to iterate on the steady state of aggregate capital, not on the law of motion, as in the Krusell–Smith method.

References

Atkeson, A., Phelan, C., 1994. Reconsidering the costs of business cycles with incomplete markets. NBER Working Paper Series 4719.

Beaudry, P., Pages, C., 2001. The cost of business cycles and the stabilization value of unemployment insurance. Eur. Econ. Rev. 45 (8), 1545–1572.

Bell, D.N.F., Blachflower, D.G., 2011. Young people and the great recession. Oxf. Rev. Econ. Policy 27 (2), 241–267.

Bruno, G.S., Choudhry, M.T., Marelli, E., Signorelli, M., 2014. Youth unemployment: key determinants and the impact of crisis. In: Sciulli, D., Malo, M.A. (Eds.), Disadvantaged Workers. Empirical Evidence and Labour Policies, Springer.

Burgess, S., Propper, C., Rees, H., Shearer, A., 2003. The class of 1981: the effects of early career unemployment on subsequent unemployment experiences. Labour Econ. 10 (3), 291–309.

Chéron, A., Hairault, J.-O., Langot, F., 2013. Life-cycle equilibrium unemployment. J. Labor Econ. 31 (4), 843–882.

CSO, 2014. Population Projection 2014–2050. Central Statistical Office, Warsaw.

Fiodra, M., Fratzscher, M., Thimann, C., 2007. Home bias in global bond and equity markets: the role of real exchange rate volatility. J. Int. Money Financ. 26 (4), 631–655.

Gomes, J.F., Greenwood, J., Rebelo, S.T., 2001. Equilibrium unemployment. J. Monet. Econ. 48 (1), 109–152.

Hairault, J.-O., Langot, F., Osotimehin, S., 2010. Matching frictions, unemployment dynamics and the cost of business cycle. Rev. Econ. Dyn. 13 (4), 759–779.

Heer, B., Maussner, A., 2009. Dynamic General Equilibrium Modelling. Computational Methods and Applications. 2nd edition, Springer, Berlin.

Hoynes, H., Miller, D.L., Schaller, J., 2012. Who suffers during recessions?. J. Econ. Perspect. 26 (3), 27–48.

Iliopulos, E., Langot, F., Sopraseuth, T., 2014. Welfare cost of fluctuations: when labor market search interacts with financial frictions. Documents de travail du Centre d'Economie de la Sorbonne 2014.42, Ftp://mse.univ-paris1.fr/pub/mse/CES2014/14042.pdf

ILO, 2012. The Youth Employment Crisis: Time for Action. International Labour Office, Geneva.

Imrohoroğlu, A., 1989. Cost of business cycles with indivisibilities and liquidity constraints. J. Polit. Econ. 97 (6), 1364–1383.

Jimeno, J.F., Rodriguez-Palenzuela, D., 2002. Youth unemployment in the OECD: demographic shifts, labour market institutions, and macroeconomic shocks. ECB Working Paper No. 155.

Jung, P., Kuester, K., 2011. The (un)importance of unemployment fluctuations for the welfare cost of business cycles. J. Econ. Dyn. Control. 35 (10), 1744–1768.

Kahn, L.B., 2010. The long-term labor market consequences of graduating from college in a bad economy. Labour Econ. 17 (2), 303–316.

Kawaguchi, D., Murao, T., 2012. Who bears the cost of the business cycle? Labor-market institutions and volatility of the youth unemployment rate. IZA J. Labor Policy 1 (1), 1–22.

Krusell, P., Mukoyama, T., Şahin, A., Smith, A.A., 2009. Revisiting the welfare effects of eliminating business cycle. Rev. Econ. Dyn. 12 (3), 393–402.

Krusell, P., Smith, A.A., 1998. Income and wealth heterogeneity in the macroeconomy. J. Polit. Econ. 106 (5), 867–896.

Krusell, P., Smith, A.A., 1999. On the welfare effects of eliminating business cycle. Rev. Econ. Dyn. 2 (1), 245–272.

Kwiecińska, A., 2011. Increasing the retirement age —necessary conditions and impact on the replacement rate. Pension System — Problems for the Future, Social Security Office pp. 91–96.

Landais, C., Michaillat, P., Saez, E., 2010. Optimal unemployment insurance over the business cycle. NBER Working Paper Series 16526,

Lucas, R.E.J., 1987. Models of Business Cycle. Basil Blackwell, New York.

Maliar, L., Maliar, S., Valli, F., 2010. Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm. J. Econ. Dyn. Control. 34 (1), 42–49.

Michelacci, C., Ruffo, H., 2015. Optimal life cycle unemployment insurance. Am. Econ. Rev. 105 (2), 816–859.

Mitman, K., Rabinovich, S., 2015. Optimal unemployment insurance in an equilibrium business-cycle model. J. Monet. Econ. 71, 99–118.

Mukoyama, T., Şahin, A., 2006. Costs of business cycles for unskilled workers. J. Monet. Econ. 53 (8), 2179–2193.

NBP, 2015. Households' Wealth in Poland. Report From the Pilot Survey. National Bank of Poland, Warsaw.

Petrosky-Nadeau, N., Zhang, L., 2013. .Unemployment crises. NBER Working Paper Series 19207.

Reiter, M., 2012. On the welfare costs of unemployment fluctuations. Unpublished manuscript. http://elaine.ihs.ac.at/mreiter/costfluct.pdf.

Rios-Rull, J.-V., 1994. On the quantitative importance of market completeness. J. Monet. Econ. 34 (3), 463–496.

S., Scarpetta, A., Sonnet, T., Manfredi, 2010. .Rising youth unemployment during the crisis: how to prevent negative long-term consequences on a generation? OECD Social, Employment and Migration Working Papers, No. 106. http://www.oecd.org/employment/youthforum/44986030.pdf.

Storesletten, K., Telmer, C.I., Yaron, A., 2001. The welfare cost of business cycles revisited: finite lives and cyclical variation in idiosyncratic risk. Eur. Econ. Rev. 45 (7), 1311–1339.

van Vliet, O., Caminada, K., 2012. Unemployment replacement rates dataset among 34 welfare states 1971–2009: An update, extension and modification of Scruggs' welfare state entitlements data set. NEUJOBS Special Report No. 2, Leiden University, http://media.leidenuniv.nl/legacy/neujobs-vanvliet-%26-caminada-24-01-2012.pdf.

[4] Of course, we also have that $H_t(K) = H_{t-1}(H_1(K))$.